2-5 Wrap Up

24-677 Linear Control Systems

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Plan for today

- A review of what we have learned in this course
- Potential next steps
- My journey in research

Review of this course

Dynamic Modeling and Analysis

- Module 1-1: Linear Dynamics Modeling
- Module 1-2: Solving Linear Dynamics
- Module 1-3: Controllability and Observability
- Module 1-4: Realization (State Space vs Transfer Fun.)
- Module 1-5: Stability
- Each topic has a homework

⇒ **Midterm**

Design Methods for Control

- Module 2-1: Feedback Control (Pole Placement)
- Module 2-2: Introduction to Optimal Control (LQR, MPC)
- Module 2-3: Introduction to Stochastic Control (Kalman Filter)
- Module 2-4: Introduction to Adaptive Control (MRAC)
- Module 2-5: Introduction to Control-Based Methods in Learning (if time permits)
- Sub-module 1-4 have 5 projects

State Space Representation

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}$$
$$\mathbf{y} = \mathbf{C}\mathbf{x} + \mathbf{D}\mathbf{u}$$

Rudolf Kálmán, 1930-2016

Linearization

Consider the nonlinear system:

$$\dot{\mathbf{x}} = f(\mathbf{x}, \mathbf{u}), f : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}$$

Jacobi, 1804-1851

$\bar{\mathbf{x}} \in \mathbb{R}^n$ is an **equilibrium point**:

$$\exists \bar{\mathbf{u}} \in \mathbb{R}^m, \text{s.t.} f(\bar{x}, \bar{\mathbf{u}}) = 0$$

Define the deviation variables:

$$\delta_{\mathbf{x}} = \mathbf{x} - \bar{\mathbf{x}}, \delta_{\mathbf{u}} = \mathbf{u} - \bar{\mathbf{u}} \Rightarrow x = \delta_{\mathbf{x}} + \bar{x}, \mathbf{u} = \delta_{\mathbf{u}} + \bar{\mathbf{u}}$$

$$\mathbf{A} = \frac{\partial f}{\partial \mathbf{x}}|_{\mathbf{x}=\bar{\mathbf{x}},\mathbf{u}=\bar{\mathbf{u}}} \in \mathbb{R}^{n\times n}, \mathbf{B} = \frac{\partial f}{\partial \mathbf{u}}|_{\mathbf{x}=\bar{\mathbf{x}},\mathbf{u}=\bar{\mathbf{u}}} \in \mathbb{R}^{n\times m}, \frac{\partial f}{\partial \mathbf{x}} \text{ is called "\textbf{Jacobian}"}$$

Solutions to Linear Time Invariant State Equations

$$\begin{aligned}\dot{x} &= Ax + Bu \\ y &= Cx + Du\end{aligned} \quad \Rightarrow \begin{cases} x(t) = e^{A(t-t_0)}x(t_0) + \int_{t_0}^{t} e^{A(t-\tau)}Bu(\tau)d\tau \\ y(t) = Ce^{A(t-t_0)}x(t_0) + C\int_{t_0}^{t} e^{A(t-\tau)}Bu(\tau)d\tau + Du(t) \end{cases}$$

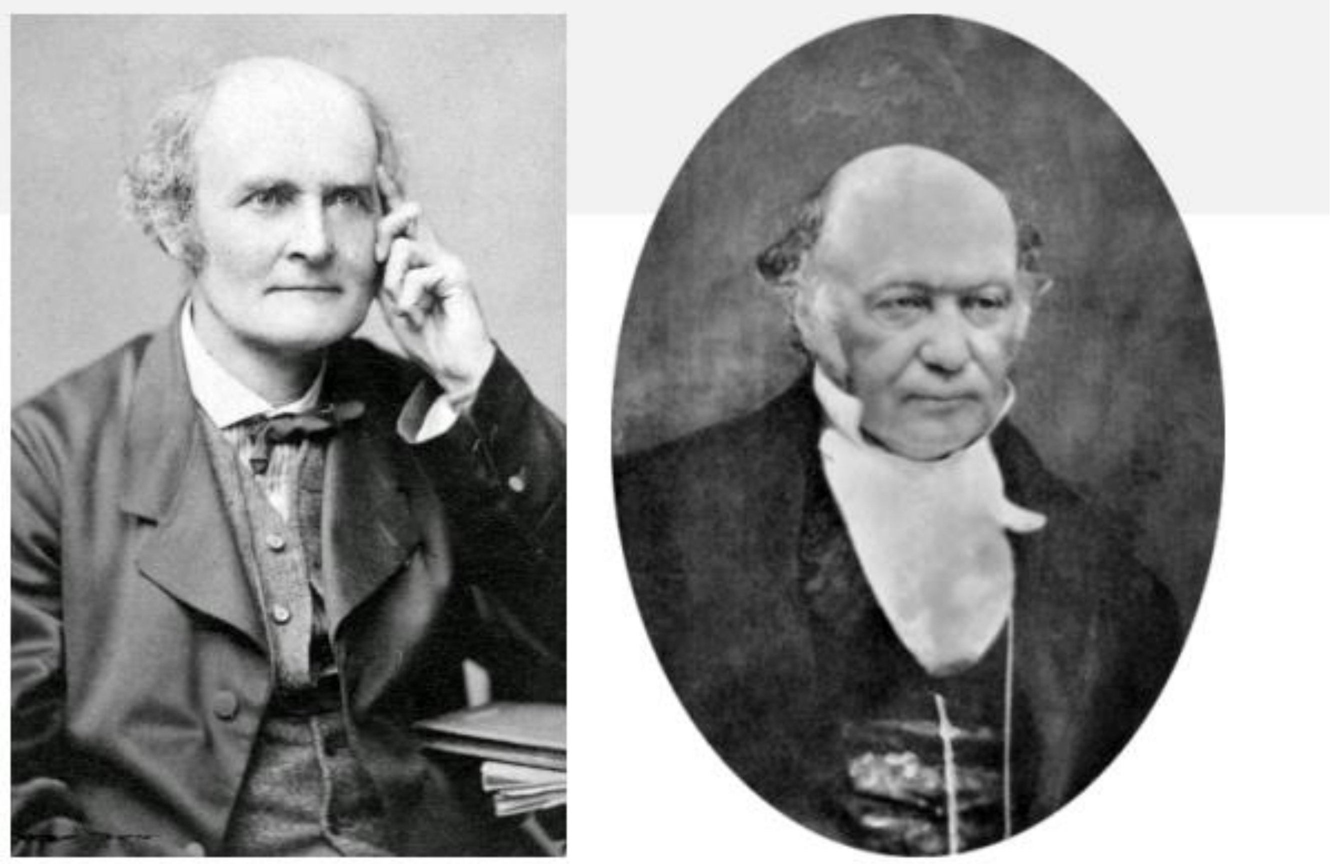
CT-LTI

Dicretization: $\Downarrow A_d = e^{AT}, B_d = \int_0^T e^{A\tau}d\tau B = A^{-1}\left(A_d - I\right)B$

$$\begin{aligned}x(k+1) &= A_d x(k) + B_d u(k) \\ y(k) &= Cx(k) + Du(k)\end{aligned} \quad \Rightarrow \begin{cases} x(k) = A_d^k x(0) + \sum_{m=0}^{k-1} A_d^{k-m-1}B_d u(m) \\ y(k) = CA_d^k x(0) + \sum_{m=0}^{k-1} CA_d^{k-m-1}B_d u(m) + Du(k) \end{cases}$$

DT-LTI

Cayley Hamilton Theorem



Cayley 1821-1895
Hamilton 1805-1865

Given $\mathbf{A} \in \mathbb{R}^{n\times n}$ with characteristic polynomial
$\Delta(\lambda) = \det(\lambda I - \mathbf{A}) = \lambda^n + \alpha_{n-1}\lambda^{n-1} + \ldots + \alpha_1\lambda + \alpha_0$. Then,

$$\Delta(\mathbf{A}) = \mathbf{A}^n + \alpha_{n-1}\mathbf{A}^{n-1} + \ldots + \alpha_1\mathbf{A} + \alpha_0\mathbf{I} = \mathbf{0}$$

That is, $\mathbf{A}$ satisfies its own characteristic equation.

$$\mathbf{A}^n = -\alpha_{n-1}\mathbf{A}^{n-1} - \alpha_{n-2}\mathbf{A}^{n-2} - \ldots - \alpha_1\mathbf{A} - \alpha_0\mathbf{I}$$

Similarity Transformation

$$\hat{\mathbf{A}} = \mathbf{S}^{-1}\mathbf{A}\mathbf{S} \Leftrightarrow \mathbf{A} = \mathbf{S}\hat{\mathbf{A}}\mathbf{S}^{-1}$$

$$\mathbf{A}^k = \mathbf{S}\hat{\mathbf{A}}\mathbf{S}^{-1}\mathbf{S}\hat{\mathbf{A}}\mathbf{S}^{-1}\cdots\mathbf{S}\hat{\mathbf{A}}\mathbf{S}^{-1} = \mathbf{S}\hat{\mathbf{A}}^k\mathbf{S}^{-1}$$

$$\mathbf{A} = \mathbf{M}\mathbf{J}\mathbf{M}^{-1}$$

Jordan, 1838-1922

Controllability & Observability

Test controllability using $\mathrm{rank}(P)$

A DT LTI system is controllable $\Leftrightarrow \mathrm{rank}(P) = n$, where

$$P = [B \vdots AB \vdots \cdots \vdots A^{n-1}B]$$

Test observability using $\mathrm{rank}(Q)$

A DT LTI system is observable $\Leftrightarrow \mathrm{rank}(Q) = n$, where

$$Q = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \end{bmatrix}$$

Rudolf Kálmán, 1930-2016

Use Singular Value Decomposition to Check Rank

$$\mathbf{A} = \mathbf{U\Sigma V}^T = \begin{bmatrix} \\ \mathbf{u}_1 & \cdots & \mathbf{u}_r & \cdots & \mathbf{u}_m \\ \\ \end{bmatrix} \left[\begin{array}{ccc|c} \sigma_1 & & & \\ & \ddots & & \mathbf{0} \\ & & \sigma_r & \\ \hline & \mathbf{0} & & \mathbf{0} \end{array}\right] \begin{bmatrix} \\ \mathbf{v}_1 & \cdots & \mathbf{v}_r & \cdots & \mathbf{v}_n \\ \\ \end{bmatrix}^T$$

Jordan, 1838-1922

Realization

$$G(s) = C(sI - A)^{-1}B + D$$

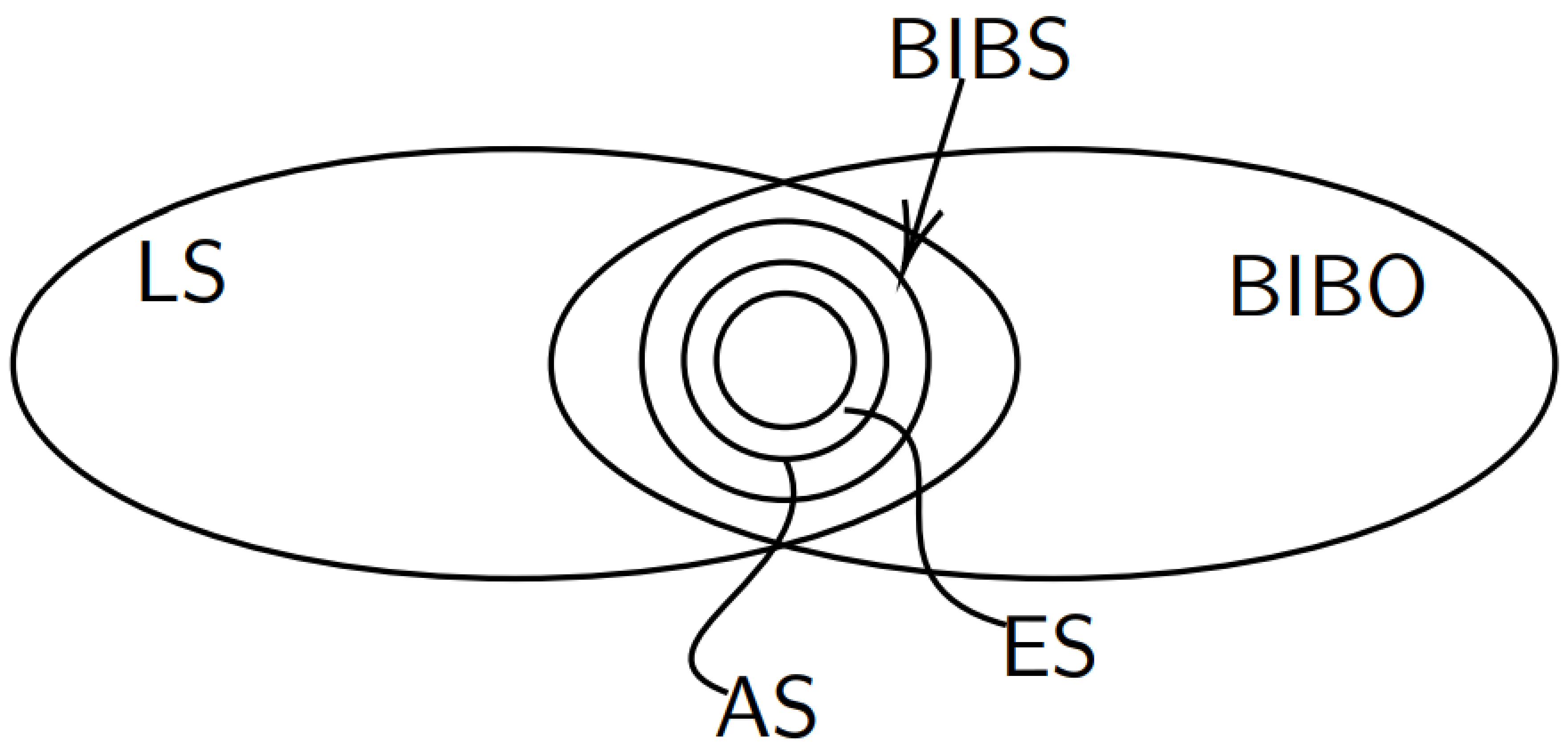
$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}$$
$$\mathbf{y} = \mathbf{C}\mathbf{x} + \mathbf{D}\mathbf{u}$$

Kalman Decomposition

$$\dot{x}_{co} = A_{co}x_{co} + B_{co}u$$
$$y = C_{co}x_{co} + Du$$

Rudolf Kálmán, 1930-2016

Stability



Lyapunov 1857-1918

PID (P1)

PID Control

- Algorithms: $u(t) = K_p e(t) + K_i \int_0^t e(\tau)\, d\tau + K_d \frac{de(t)}{dt}$
- Transfer function: $L(s) = K_p + K_i/s + K_d s$

Minorsky, 1885-1970

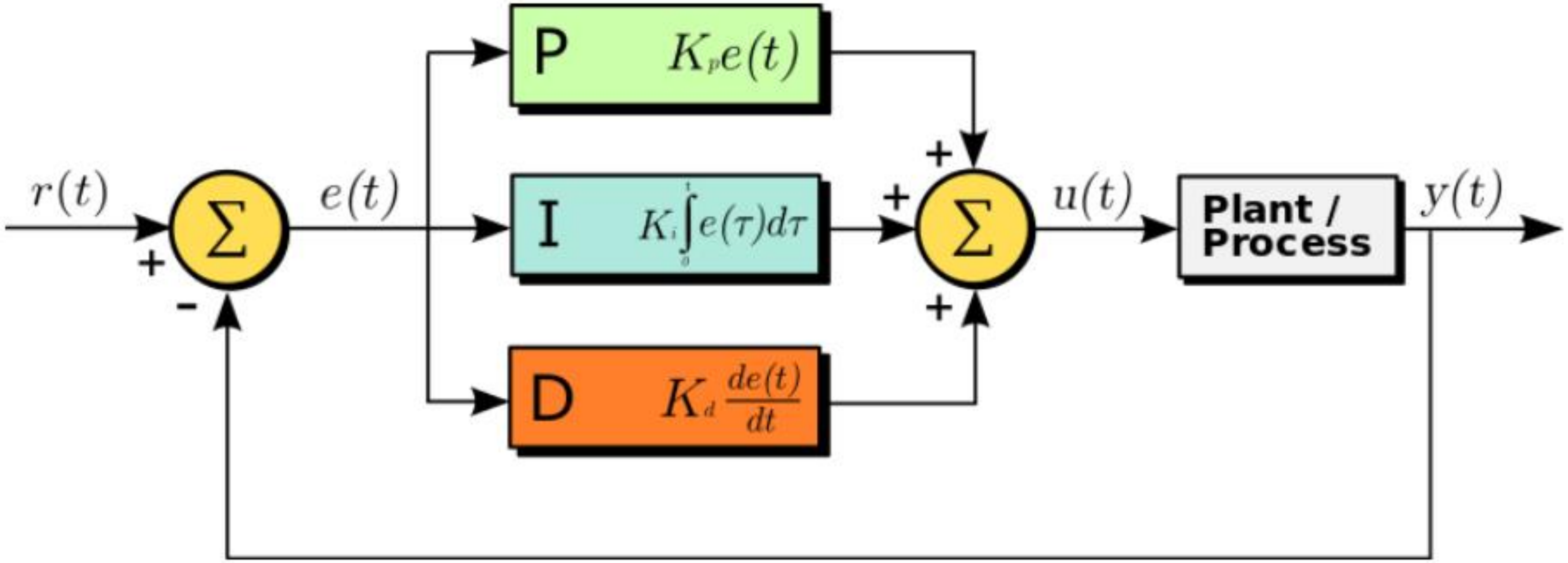


image Courtesy a Arturo Urquizo

Pole Placement (P2)

on
Sylvester, 1814-1897

$$u = K\tilde{x}$$

$$\Rightarrow \dot{x} = Ax + BK\tilde{x}, y = Cx + Du$$

$$\dot{\tilde{x}} = A\tilde{x} + Bu + L(y - \tilde{y})$$

$$\tilde{y} = C\tilde{x} + DK\tilde{x}$$

$$\Rightarrow \dot{\tilde{x}} = A\tilde{x} + BK\tilde{x} + L(Cx + DK\tilde{x} - (C\tilde{x} + DK\tilde{x}))$$

$$= (A - LC + BK)\tilde{x} + LCx$$

- Now stack $\begin{bmatrix} \dot{x} \\ \dot{\tilde{x}} \end{bmatrix} = \begin{bmatrix} A & BK \\ LC & A - LC + BK \end{bmatrix} \begin{bmatrix} x \\ \tilde{x} \end{bmatrix}$

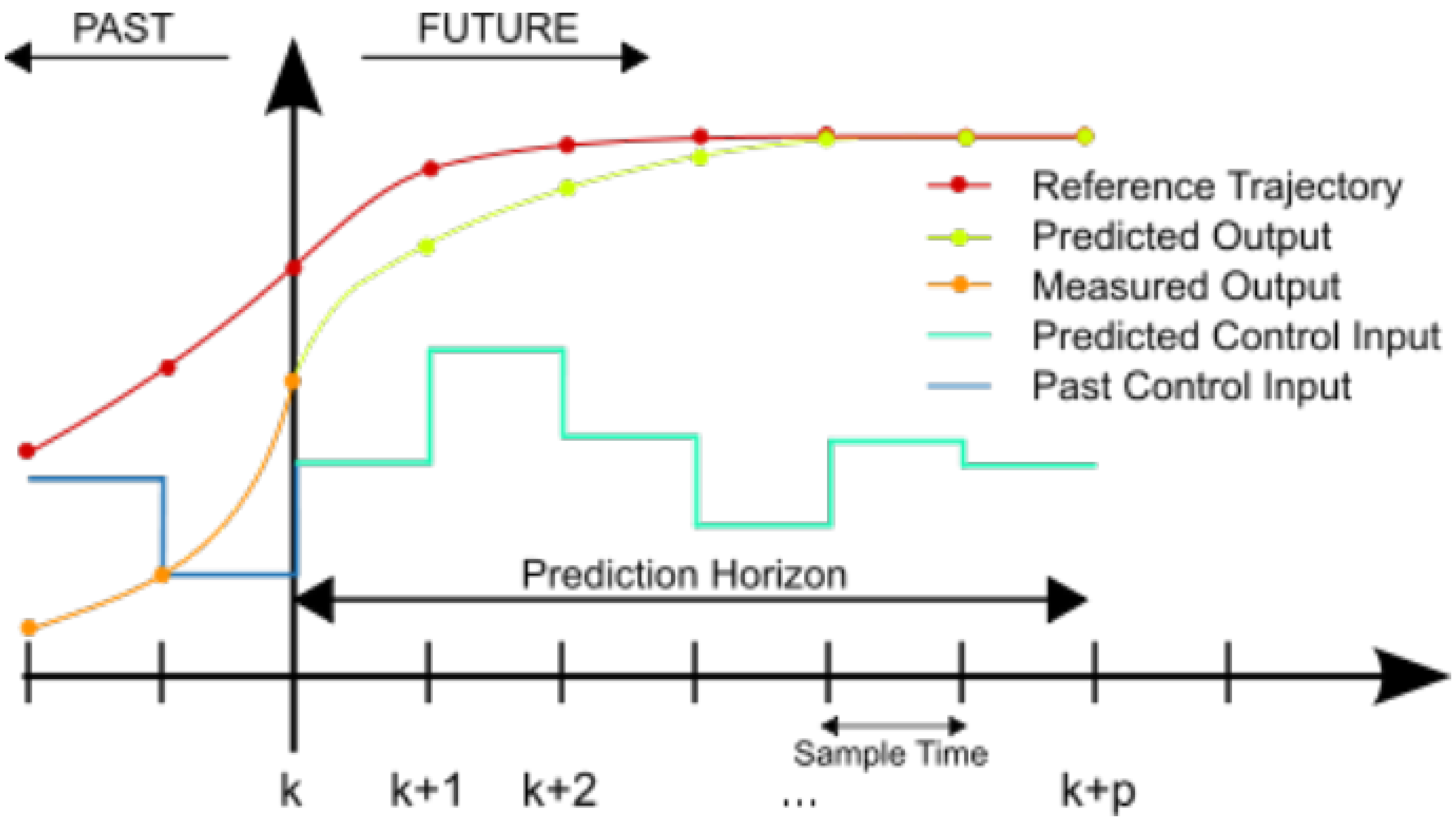
$$\begin{bmatrix} \dot{x} \\ \dot{e} \end{bmatrix} = \begin{bmatrix} A + BK & -BK \\ 0 & A - LC \end{bmatrix} \begin{bmatrix} x \\ e \end{bmatrix}$$

Optimal Control (P3)

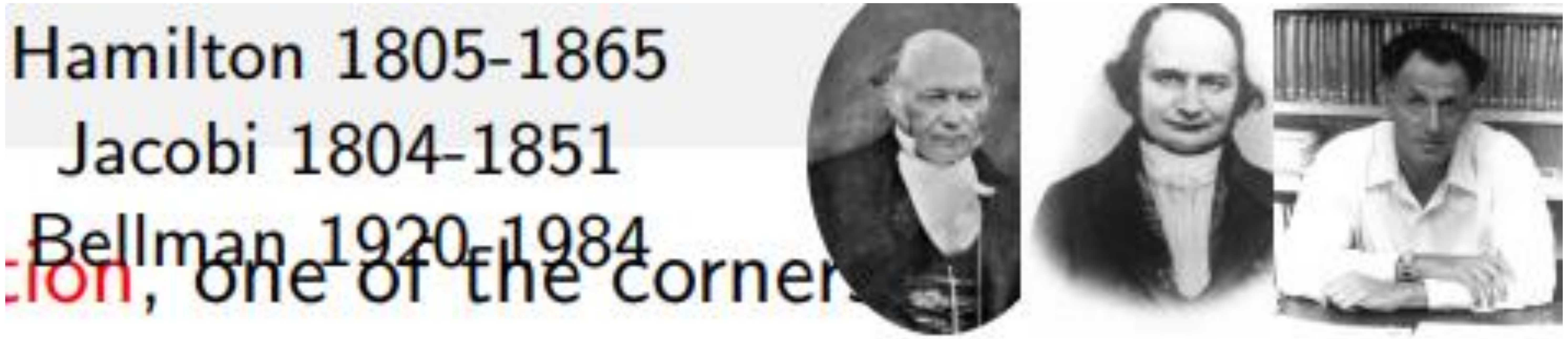
Bellman, 1920-1984

1. Finite Horizon Discrete-Time Linear Quadratic Regulator
2. Model Predictive Control
3. Infinite Horizon Discrete-Time Linear Quadratic Regulator
4. Feedforward In Optimal Control
5. Finite Horizon Continuous-Time Linear Quadratic Regulator (if time permits)
6. Infinite Horizon Continuous-Time Linear Quadratic Regulator

LQR, MPC


PAST
FUTURE
Reference Trajectory
Predicted Output
Measured Output
Predicted Control Input
Past Control Input
Prediction Horizon
Sample Time
k
k+1
k+2
...
k+p

Hamilton-Jacobi-Bellman Equation



$$\frac{\partial J^*}{\partial t} + \min_u \left\{ \frac{1}{2} \left(x^T Q x + u^T R u \right) + \frac{\partial J^*}{\partial x} (Ax + Bu) \right\} = 0$$

- Define the Hamiltonian

$$H\left(x, u, J^*, t\right) \equiv \frac{1}{2} \left(x^T Q x + u^T R u \right) + \frac{\partial J^*}{\partial x} (Ax + Bu)$$

Riccati Equation

Discrete-time Difference Riccati Equation	Continuous-time Differential Riccati Equation
$S_k = A^T S_{k+1} A - A^T S_{k+1} B (R + B^T S_{k+1} B)^{-1} B^T S_{k+1} A + Q$	$\dot{P}(t) = -Q + P(t) B R^{-1} B^T P(t) - P(t) A - A^T P(t)$
Discrete-time Algebraic Riccati Equation (DARE)	**Continuous-time Algebraic Riccati Equation (CARE)**
$S = A^T S A - A^T S B (R + B^T S B)^{-1} B^T S A + Q$	$A^T P + P A - P B R^{-1} B^T P + Q = 0$

Rudolf Kálmán, 1930-2016

Kalman Filter, EKF, SLAM (P4)

Initiate with $\hat{\mathbf{x}}_{k-1|k-1}$ and $\mathbf{P}_{k-1|k-1}$

Predict
(P-1) Predict the state
$\hat{\mathbf{x}}_{k\|k-1} = \mathbf{A}\hat{\mathbf{x}}_{k-1\|k-1} + \mathbf{B}\mathbf{u}_k$
(P-2) Predict the error covariance
$\mathbf{P}_{k\|k-1} = \mathbf{A}\mathbf{P}_{k-1\|k-1}\mathbf{A}^{\mathrm{T}} + \mathbf{W}$

Take $\hat{\mathbf{x}}_{k|k-1}$ and $\mathbf{P}_{k|k-1}$

Correct
(C-1) Compute the Kalman gain
$\mathbf{L}_k = \mathbf{P}_{k\|k-1}\mathbf{C}^{\mathrm{T}}(\mathbf{C}\mathbf{P}_{k\|k-1}\mathbf{C}^{\mathrm{T}} + \mathbf{V})^{-1}$
(C-2) Update estimate with new measurement
$\hat{\mathbf{x}}_{k\|k} = \hat{\mathbf{x}}_{k\|k-1} + \mathbf{L}_k(\mathbf{y}_k - \mathbf{C}\hat{\mathbf{x}}_{k\|k-1})$
(C-3) Update the error covariance
$\mathbf{P}_{k\|k} = (\mathbf{I} - \mathbf{L}_k\mathbf{C})\mathbf{P}_{k\|k-1}$

Adaptive Control

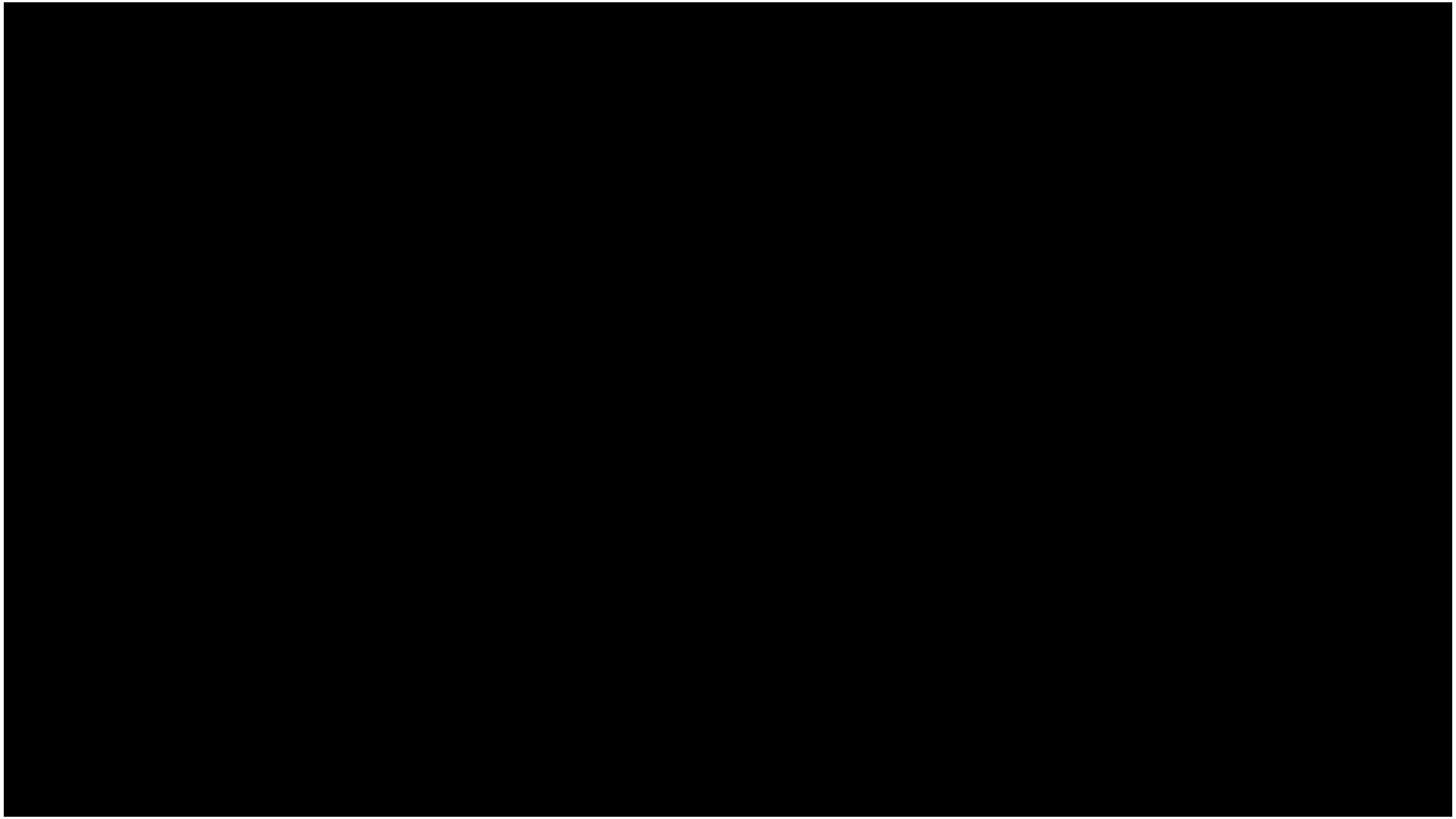
$$\dot{\hat{K}}_x = -\Gamma_x x e^T P B$$

$$\dot{\hat{K}}_r = -\Gamma_r r(t) e^T P B$$

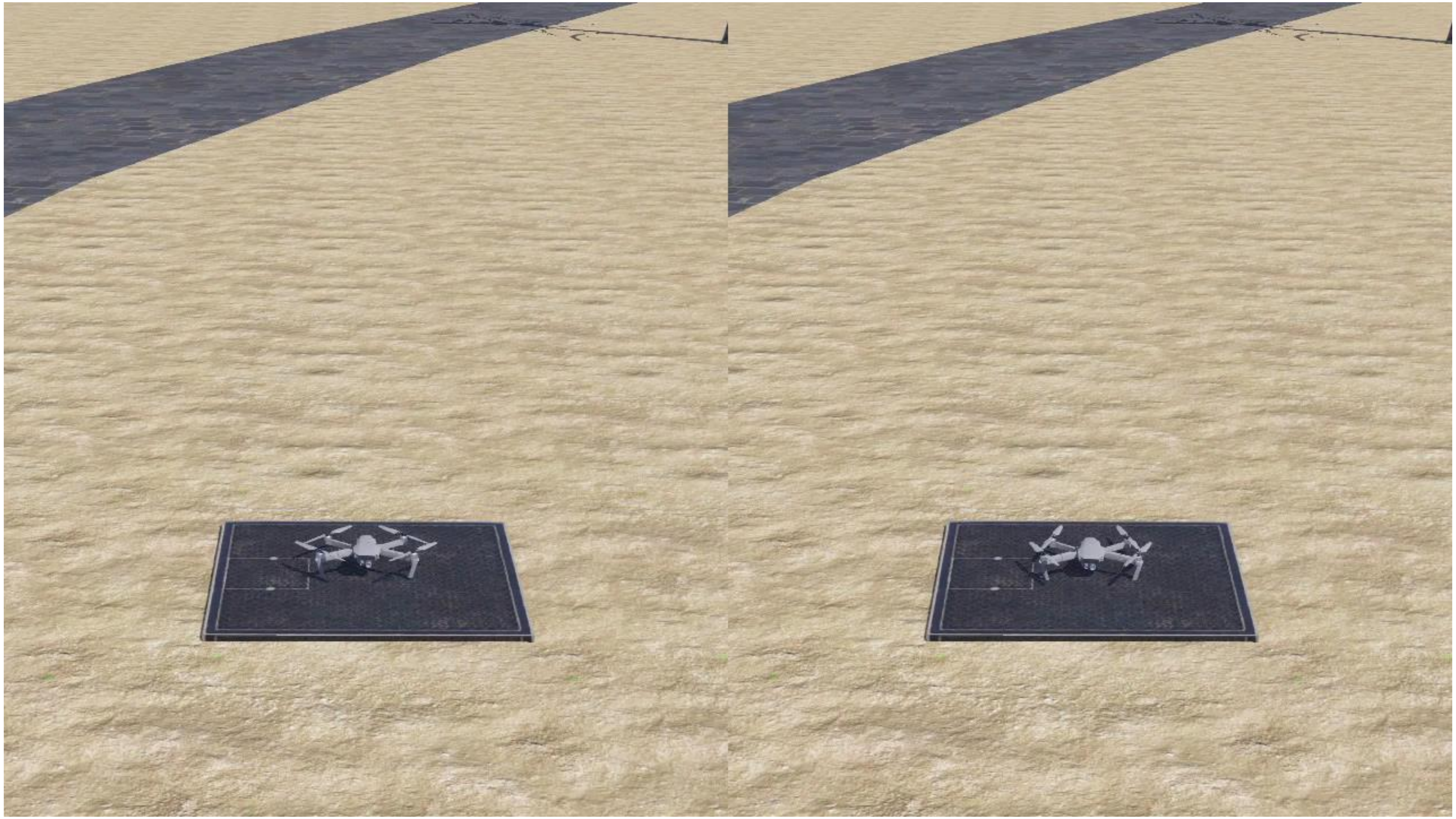
$$\dot{\hat{\Theta}} = \Gamma_\Theta \Phi(x) e^T P B$$

Lyapunov 1857-1918

Autonomous Vehicle Platform



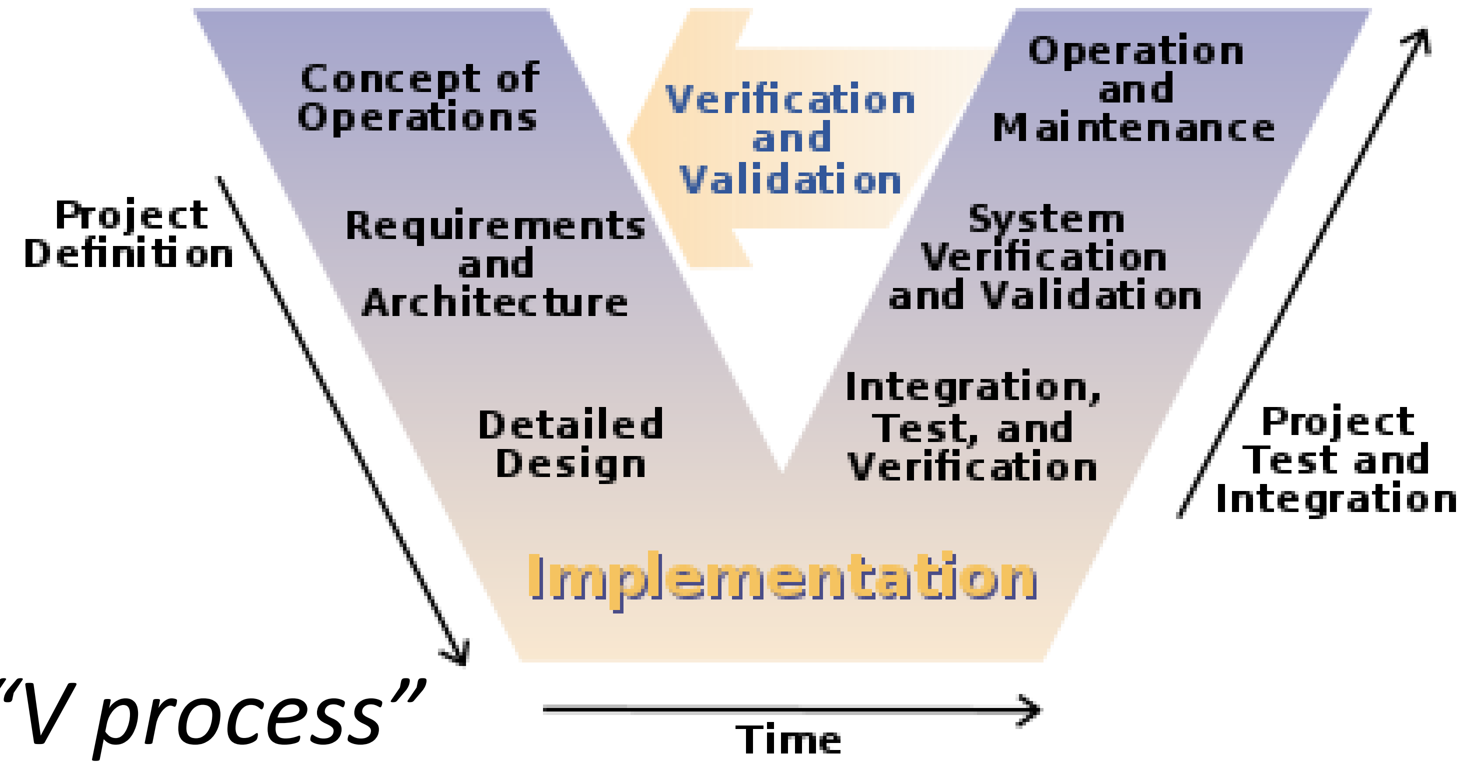
Drone Platform



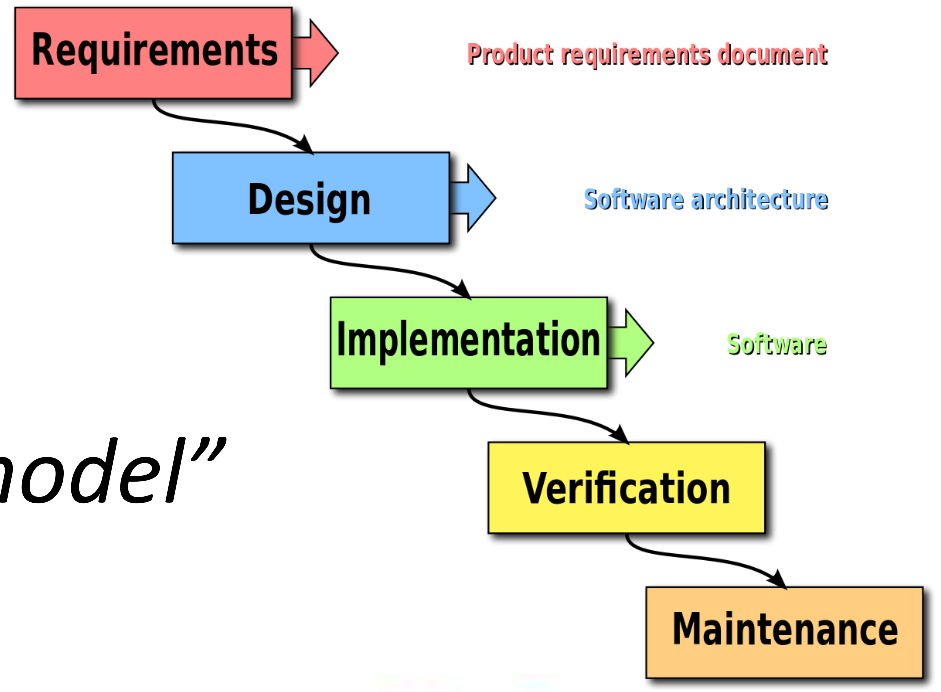
Future Directions

- More theories and practices on control
- Expertise on particular applications
  - SLAM, path planning, non-linear control, optimal control
- Control + AIML
  - Model based + Model free
  - Trustworthy AI autonomy

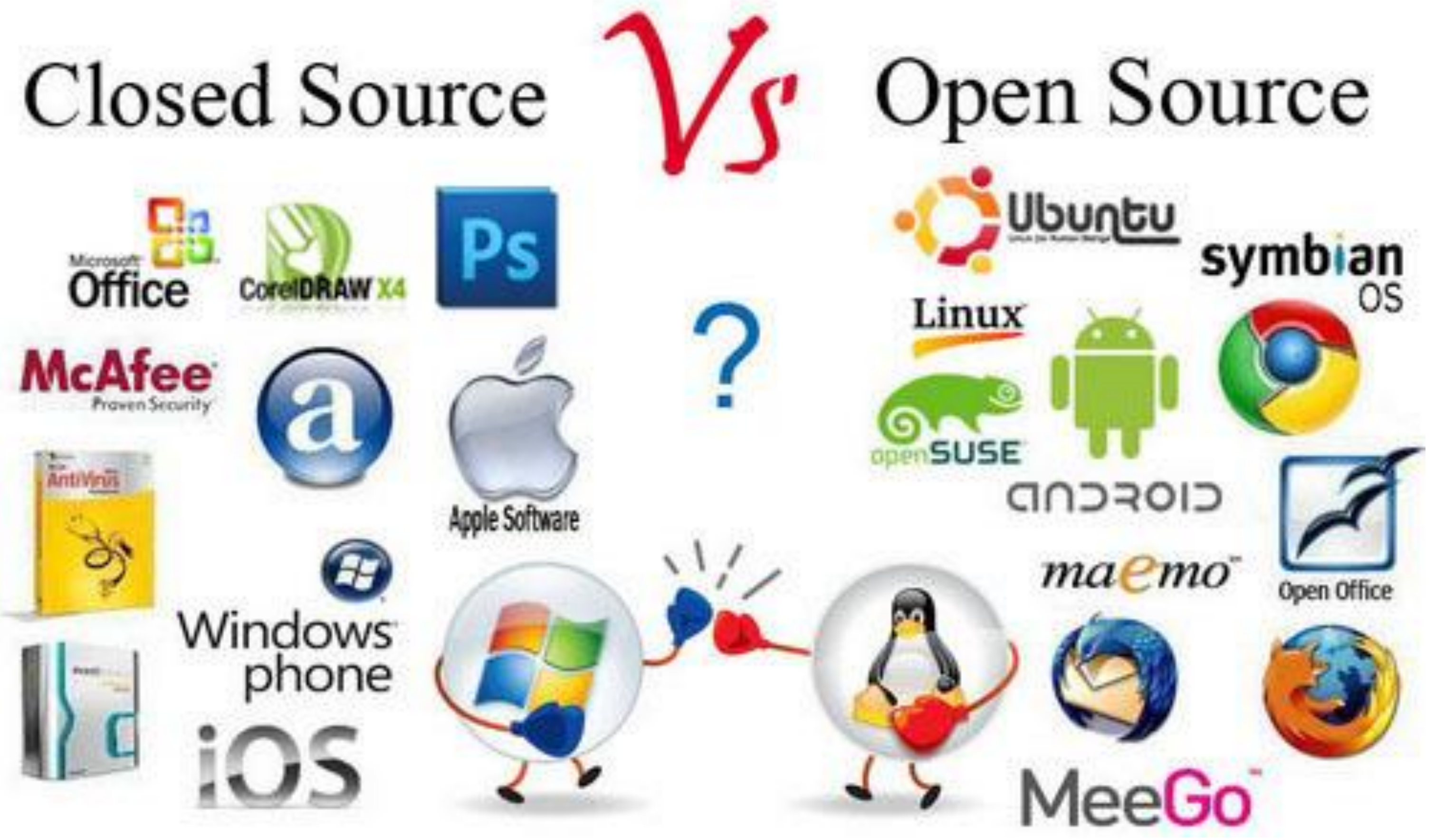
We are on the cusp to revolute the way to make machines



"V process"
Wikipedia



"Waterfall model"



Connected

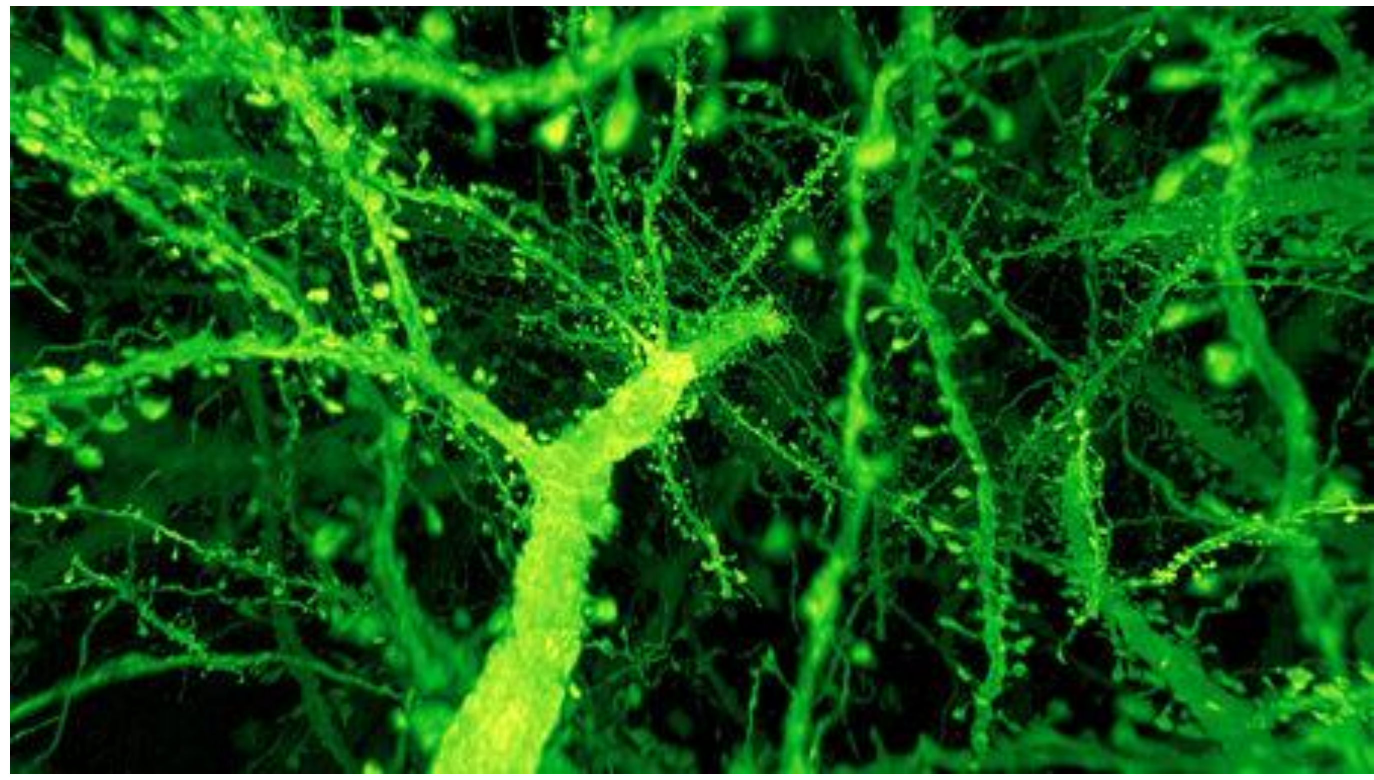
By complex structures

Evolving

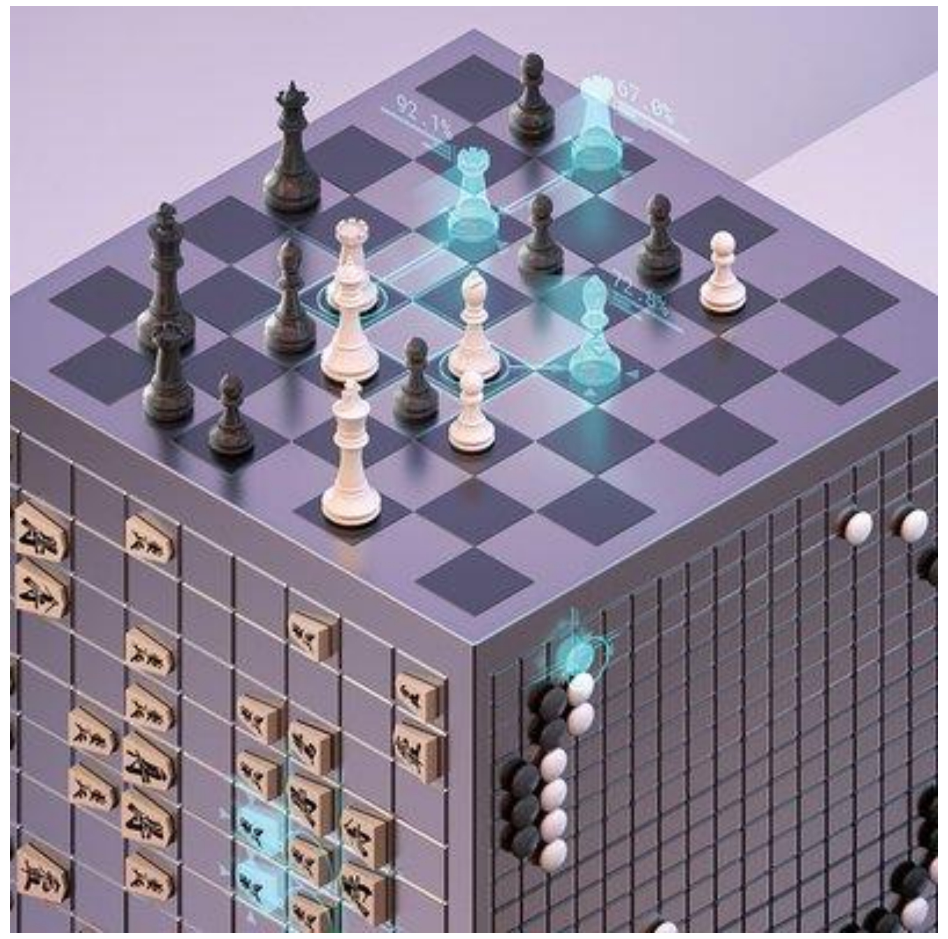
In a self-supervised way

Sharing

With blackboxes and uncertainty



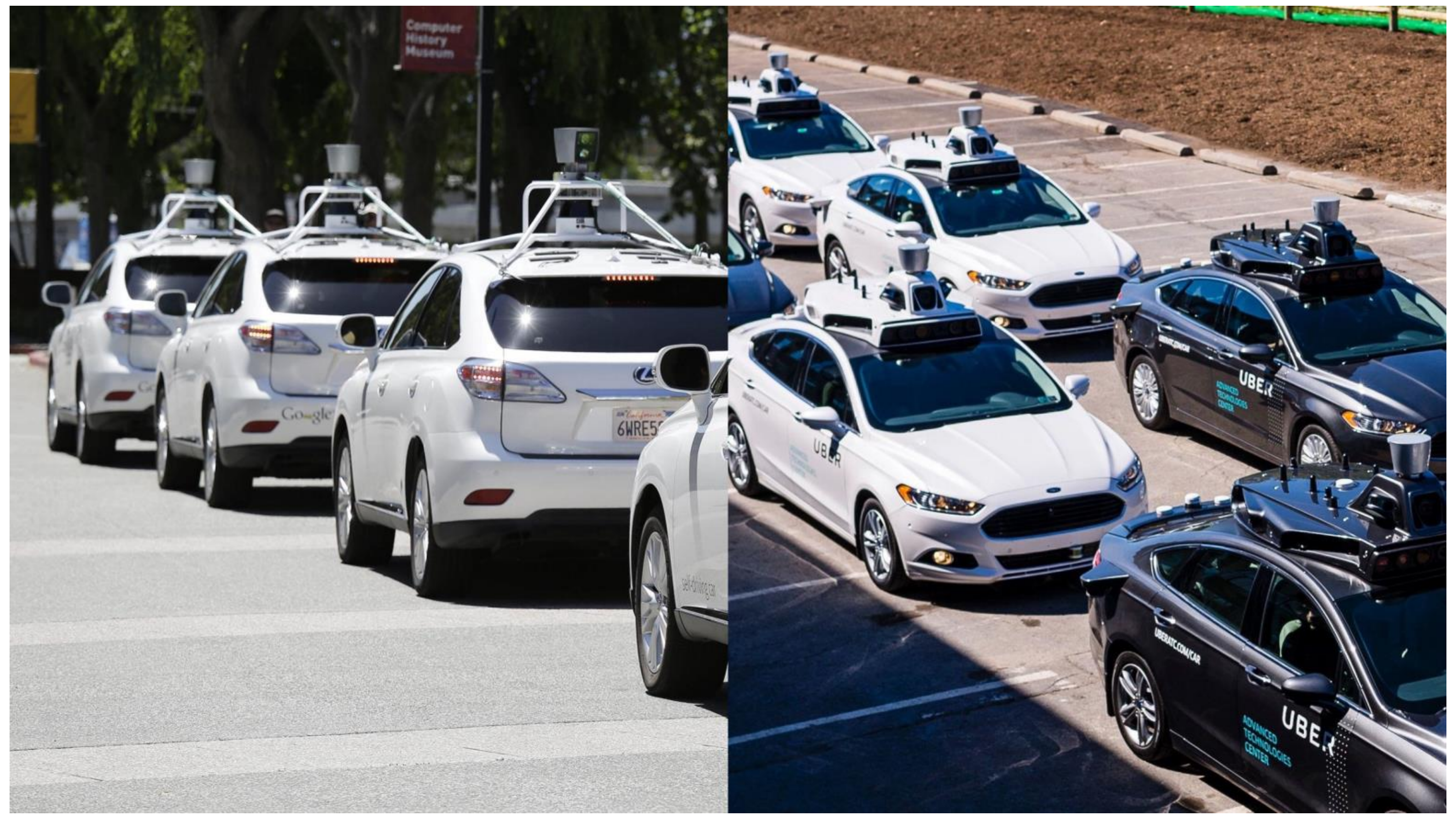
Neural Network
[Science, 2019]

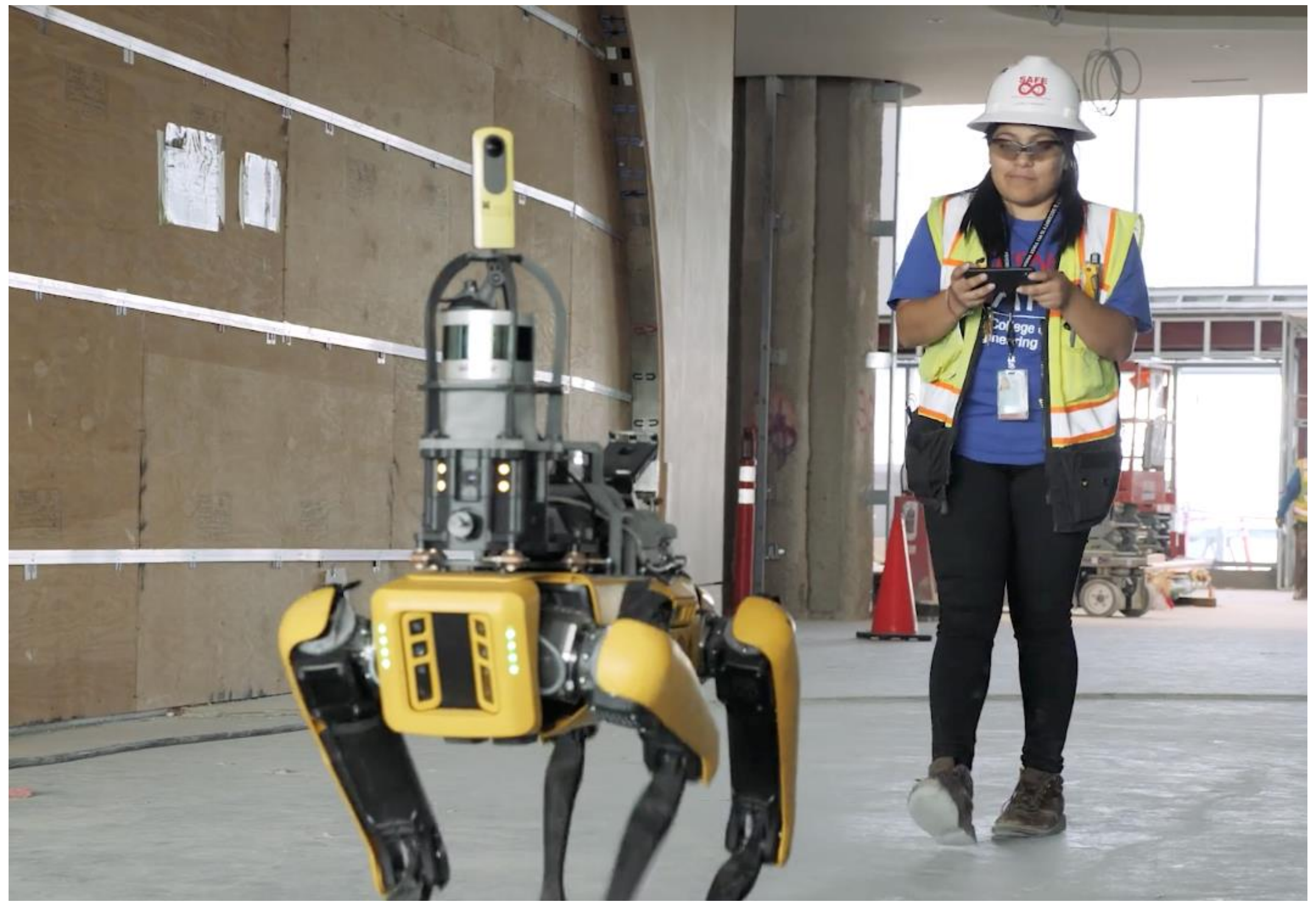


Reinforcement Learning
[Science, 2018]

Open Code/data
[Science, 2017]

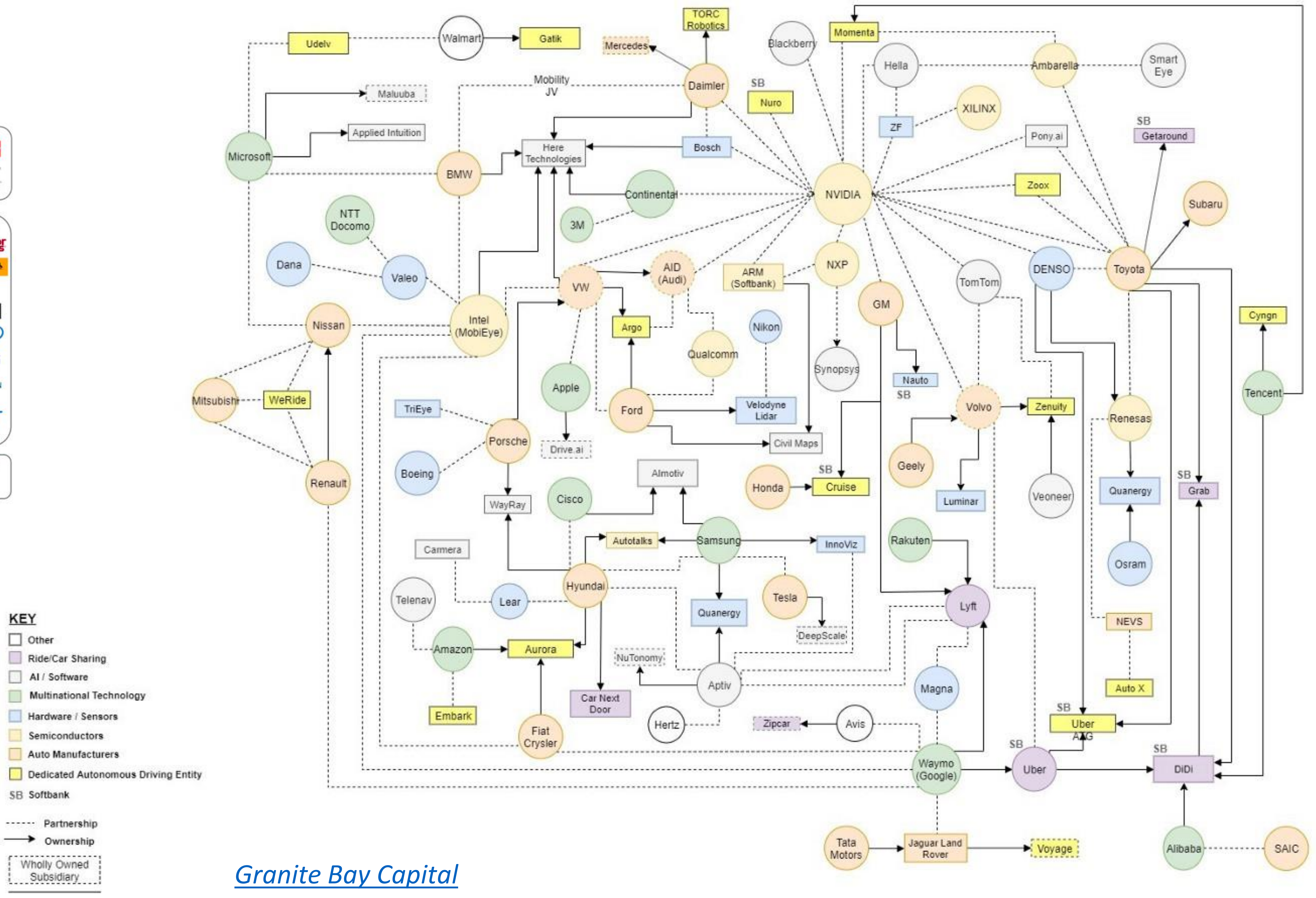
AI Autonomy





The Ever-Growing Industry

[Vision Systems Intelligence]



[Granite Bay Capital]

Tesla
Fatal Crash, May, 2016

Waymo
Accident, May, 2018

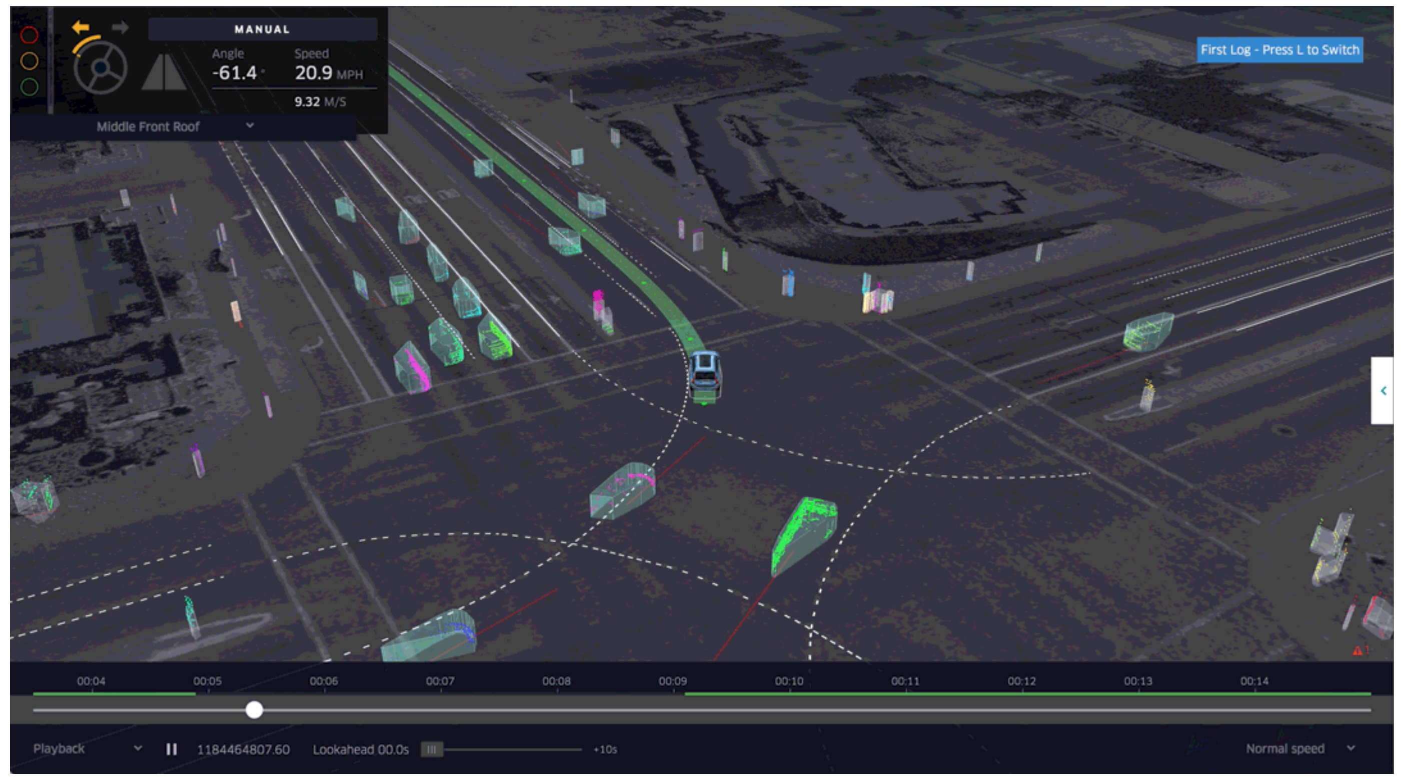
Uber
Kill a pedestrian, March, 2018

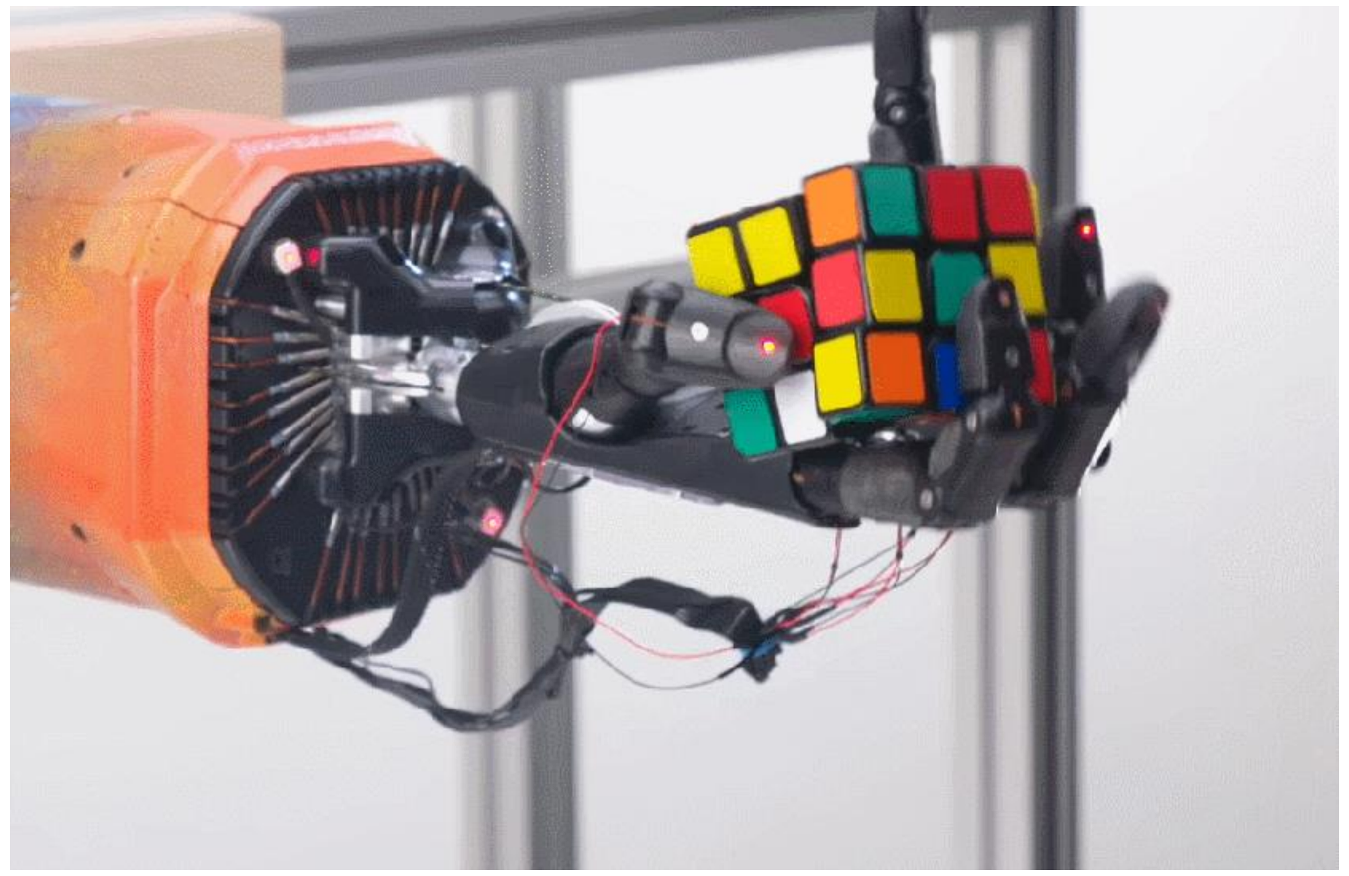
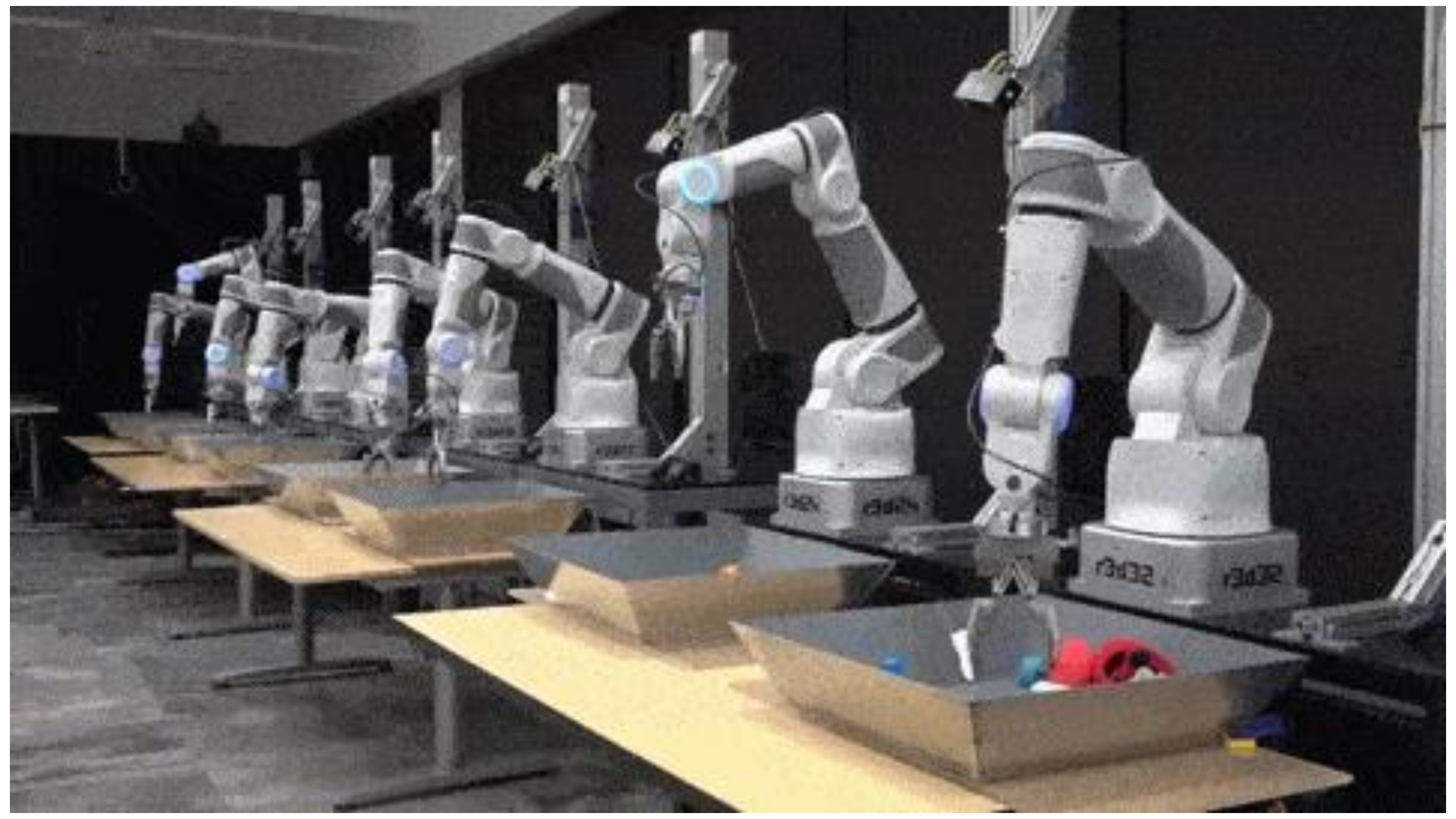
Things can go wrong
... even for the best.

How to design and assess trustworthy AI autonomy?

24-784 Special topics: Trustworthy AI Autonomy

- Spring 2021, project course
- **Topics**: adversarial learning, safe reinforcement learning, rare-event/few-shot learning, hierarchical AI, sim2real/real2sim, learning robotics implementation in the real world.
- Module 1: The Foundation
  - Review essential AIML knowledge in Trustworthy AI Autonomy
- Module 2: The Frontier
  - Read papers on recent hot topics
  - Project/guest speakers
- **Evaluation**: paper presentations, HWs, course projects
- **Pre-request**: 24-480/24-787, 24-677 or contact the instructor

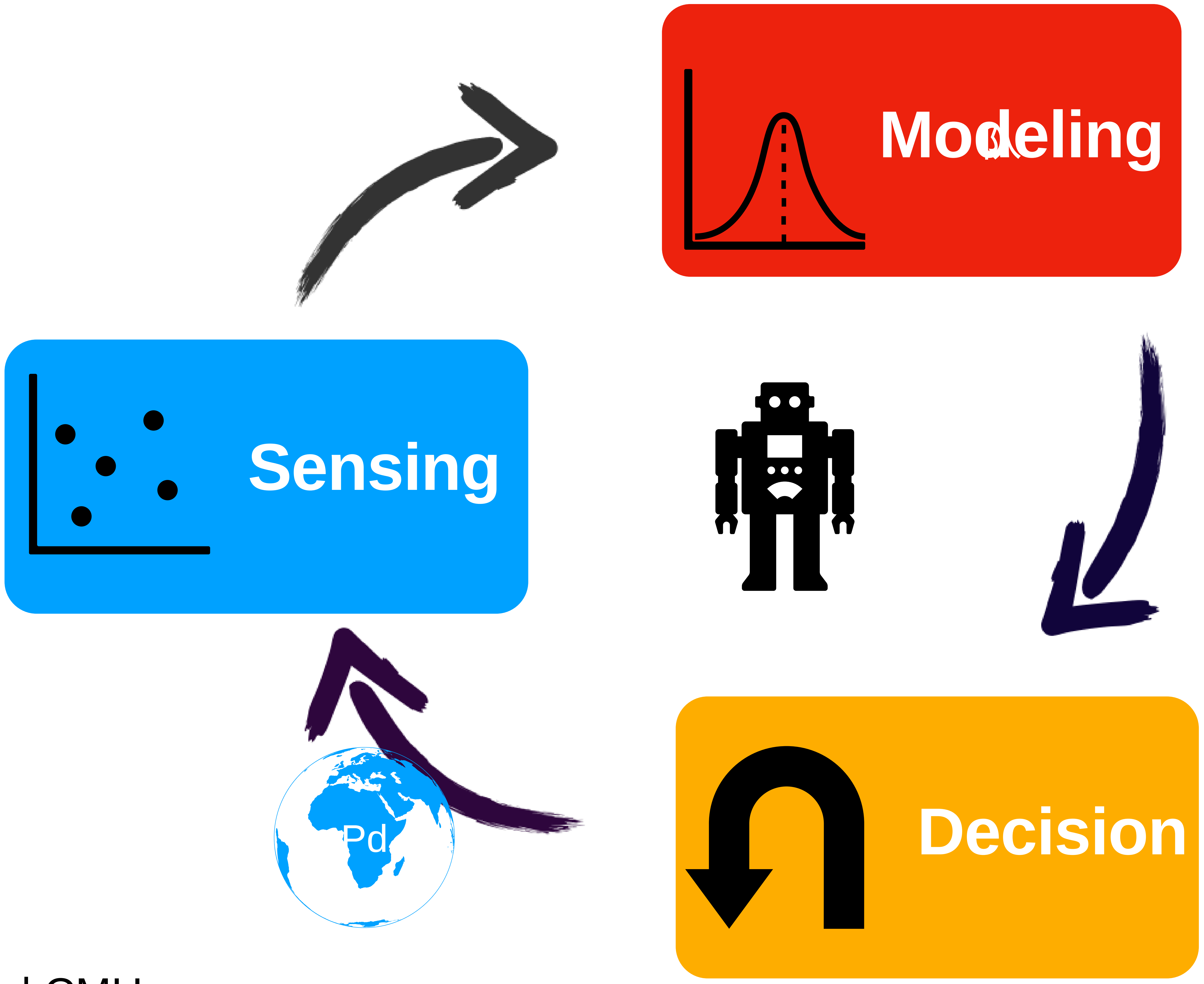


Read papers from ML:ICML/NeurIPS/ICLR
RI:CoRL/RSS/ICRA/IROS/CVPR
from academia and industry including DeepMind OpenAI

WAYMO NVIDIA amazon robotics Uber ATG Google Brain

Nutshell of AI autonomy



Some of my research

And why I care about trustworthy AI autonomy


M city
UNIVERSITY OF MICHIGAN
POLYSYNC

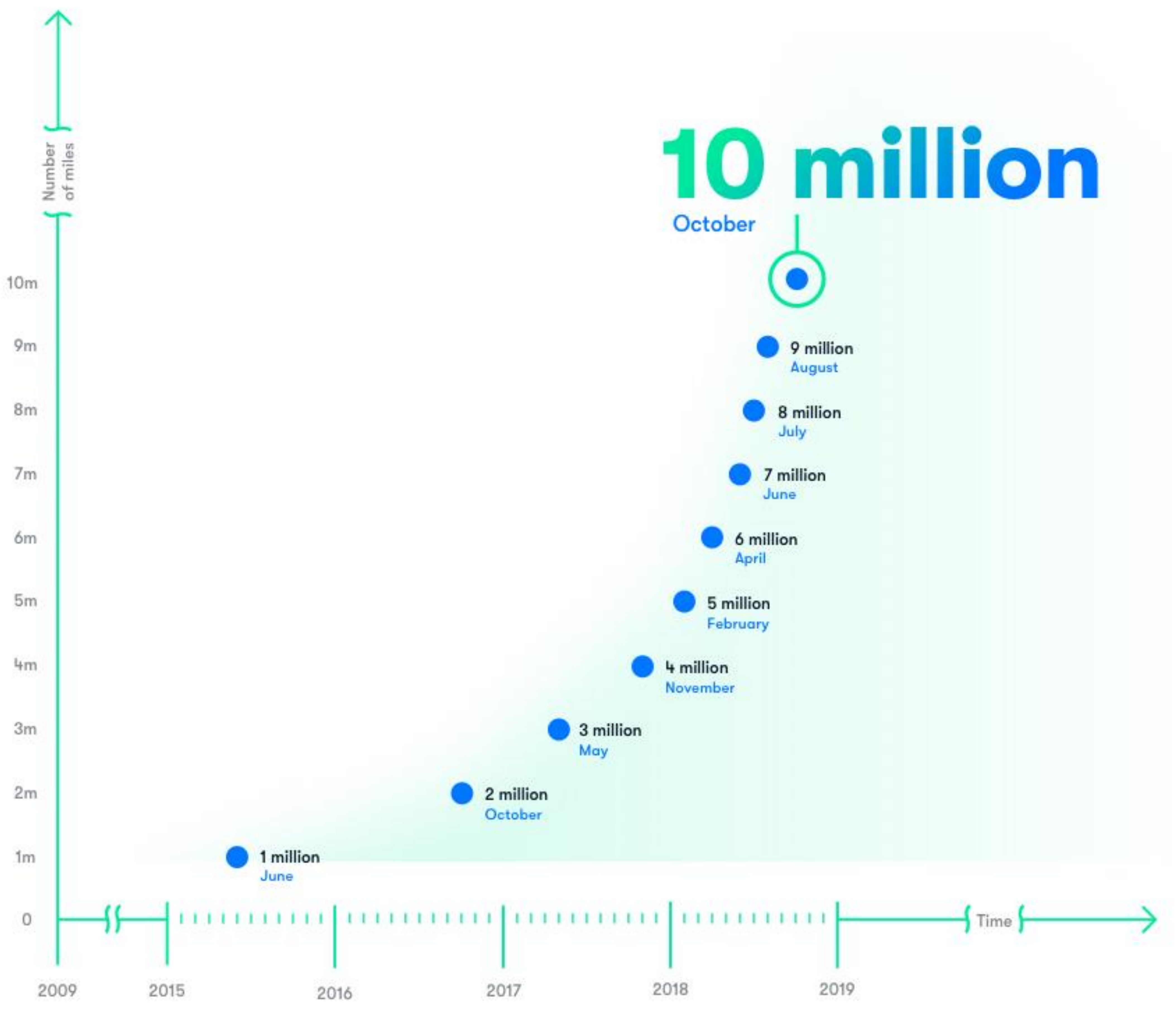
11 billion miles

To prove an AV is safer than human drivers

Rare event analysis

Nidhi Kalra, Susan M. Paddock, "How Many Miles of Driving Would It Take to Demonstrate Autonomous Vehicle Reliability? RAND report 2016

Waymo's "On the Road" program

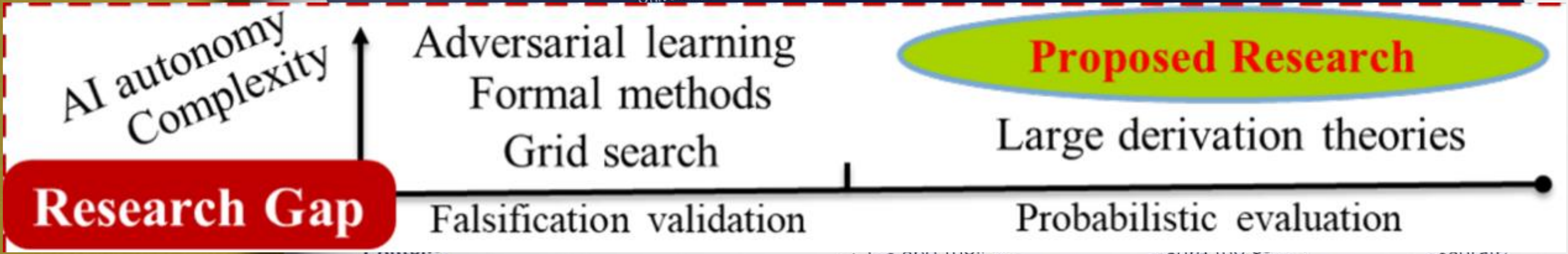


[Waymo]

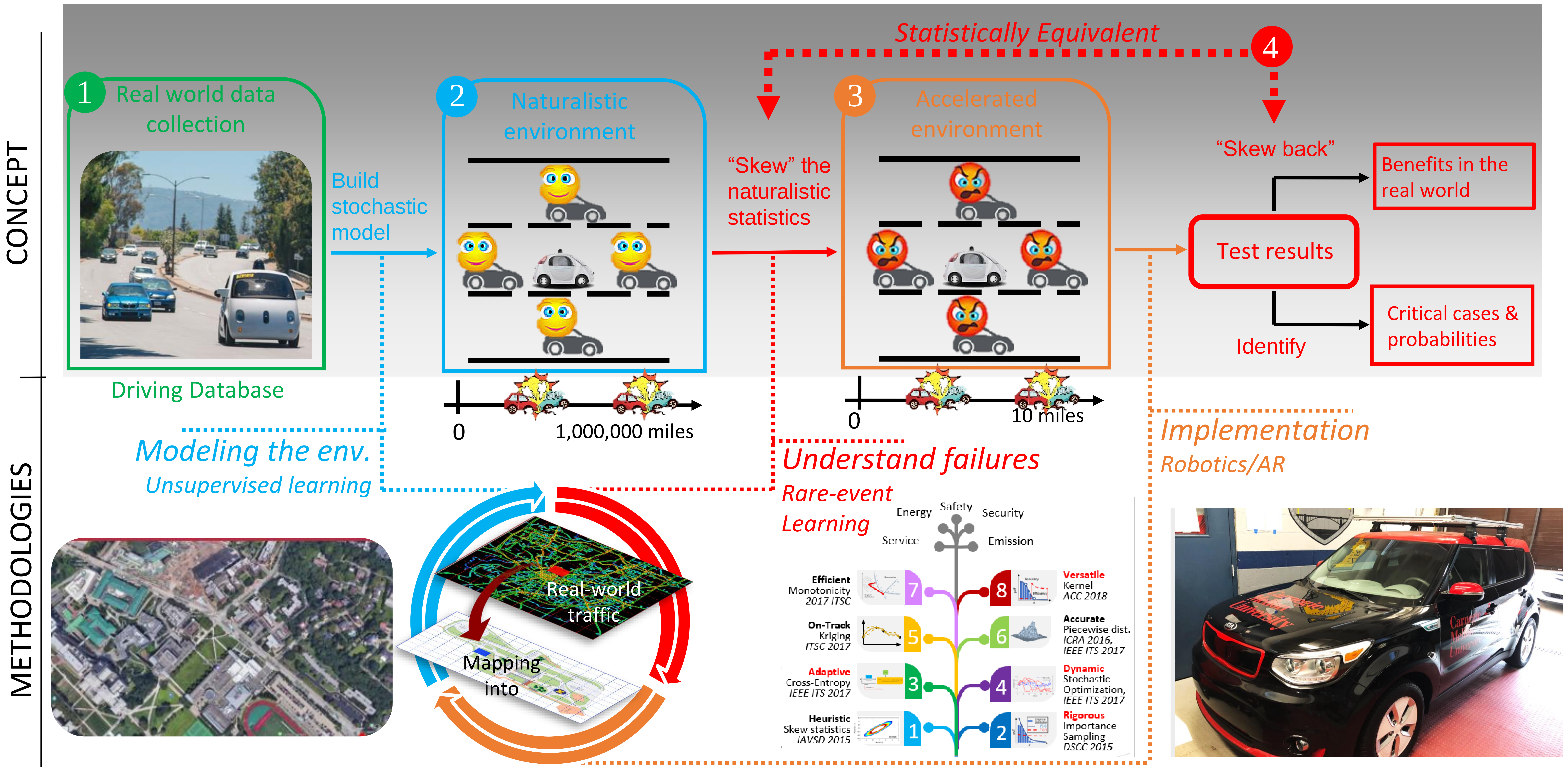
20 million miles and counting…

Forbes, January 2020


AUTOMATED VEHICLES | SIMULATION AND TESTING
M city
UNIVERSITY OF MICHIGAN
From the Lab to the Street:
Solving the Challenge of Accelerating
Automated Vehicle Testing
DING ZHAO, PhD
Assistant Research Scientist
Mechanical Engineering
University of Michigan
HUEI PENG, PhD
Director, Mcity
Roger L. McCarthy Professor
of Mechanical Engineering
University of Michigan


AI autonomy
Complexity
Adversarial learning
Formal methods
Grid search
Proposed Research
Large derivation theories
Research Gap
Falsification validation
Probabilistic evaluation

PrAE: Probabilistic Accelerated Evaluation



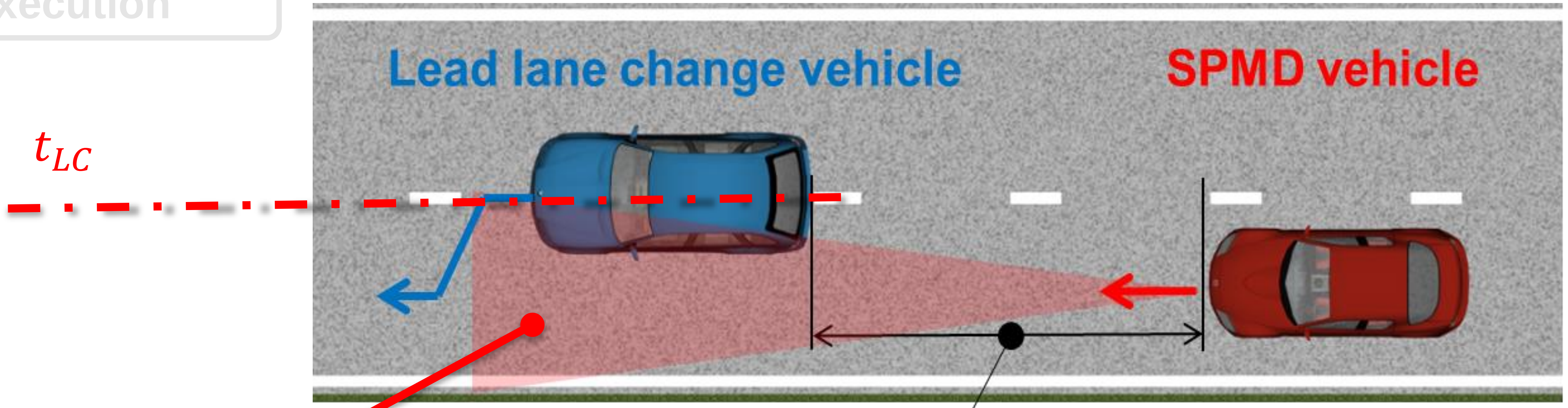
Zhao, "Accelerated Evaluation of Automated Vehicles Safety in Lane-Change Scenarios Based on Importance Sampling Techniques", IEEE ITS, 2017.

Cut-In stochastic model

- Motivation
- **Gap acceptance** → $[v_L, R_L, TTC_L]$
- Execution

Time To Collision:

$$TTC_L = -\frac{R_L}{\dot{R}_L}$$



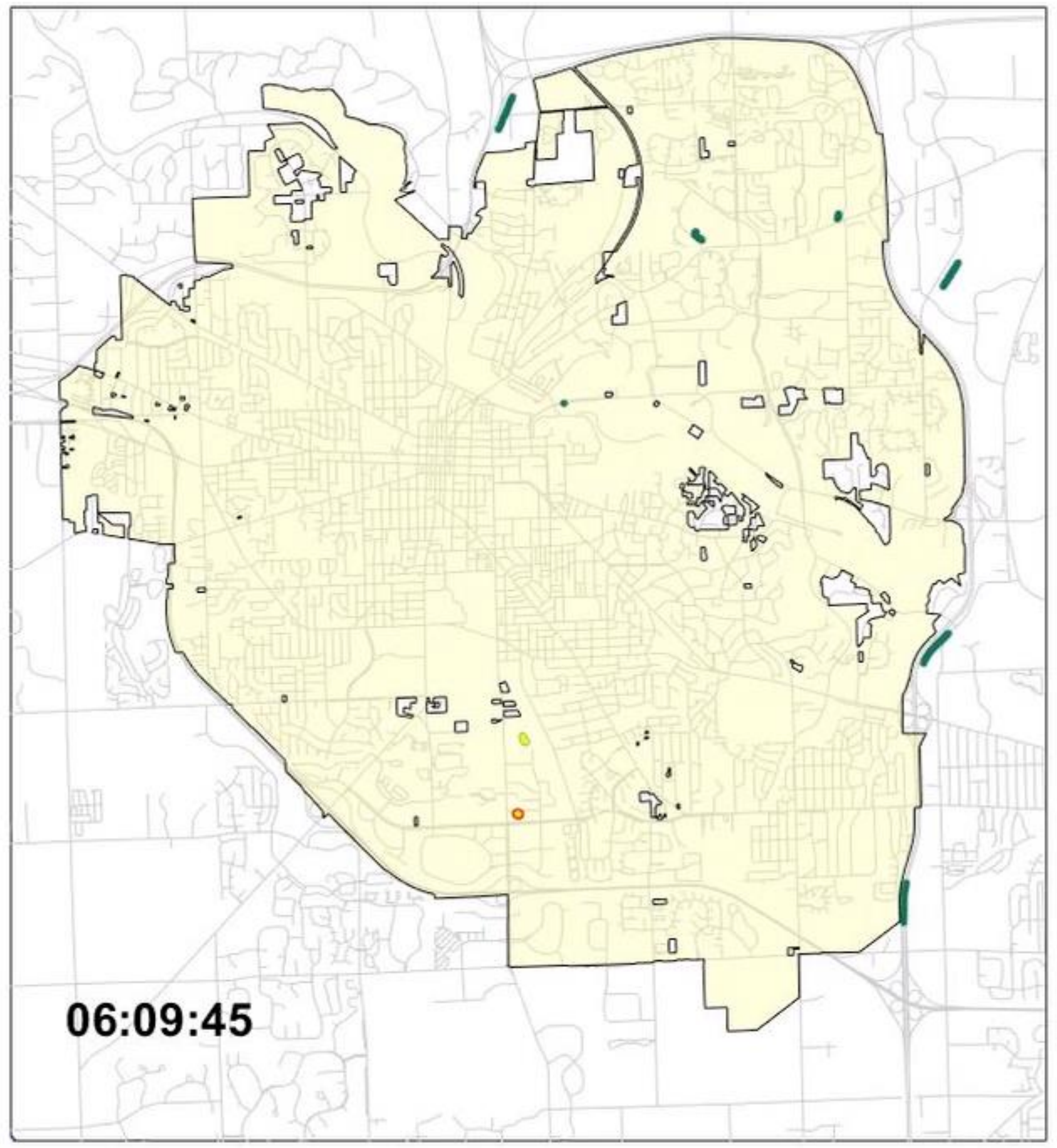
MobilEye® detection

Acceleration $a_L(t)$
Velocity $v_L(t)$
Position $D_L(t)$

Range
$R_L(t) = D_L(t) - D(t)$
Range rate
$\dot{R}_L(t) = \frac{d}{dt} R_L(t)$

Acceleration $a(t)$
Velocity $v(t)$
Position $D(t)$

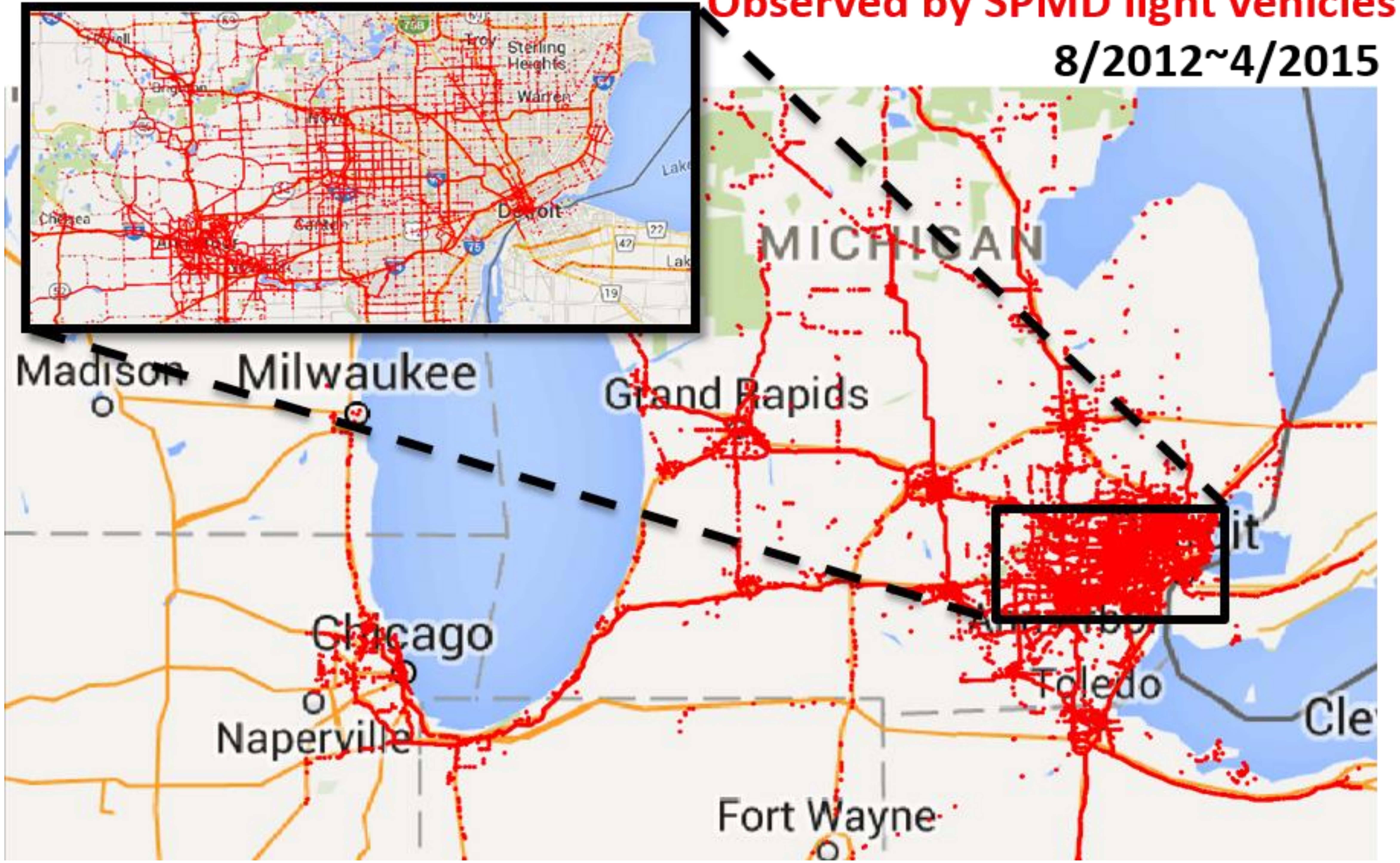
Analysis of real world driving data



One day

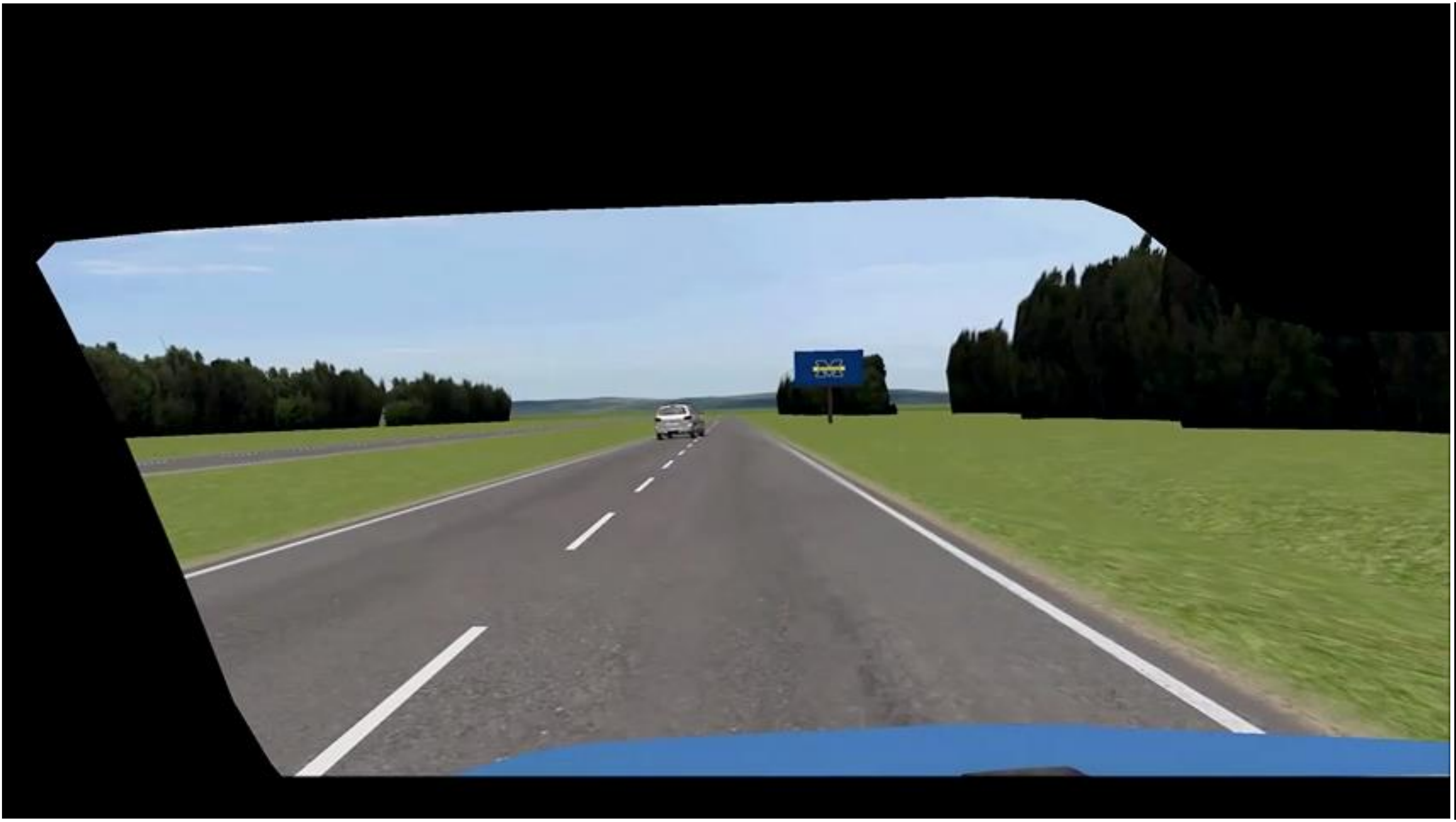
- School bus, Mobileye
- Sedan, Mobileye+GPS
- Sedan, V2V+GPS

403,581 lane change events
Observed by SPMD light vehicles
8/2012~4/2015

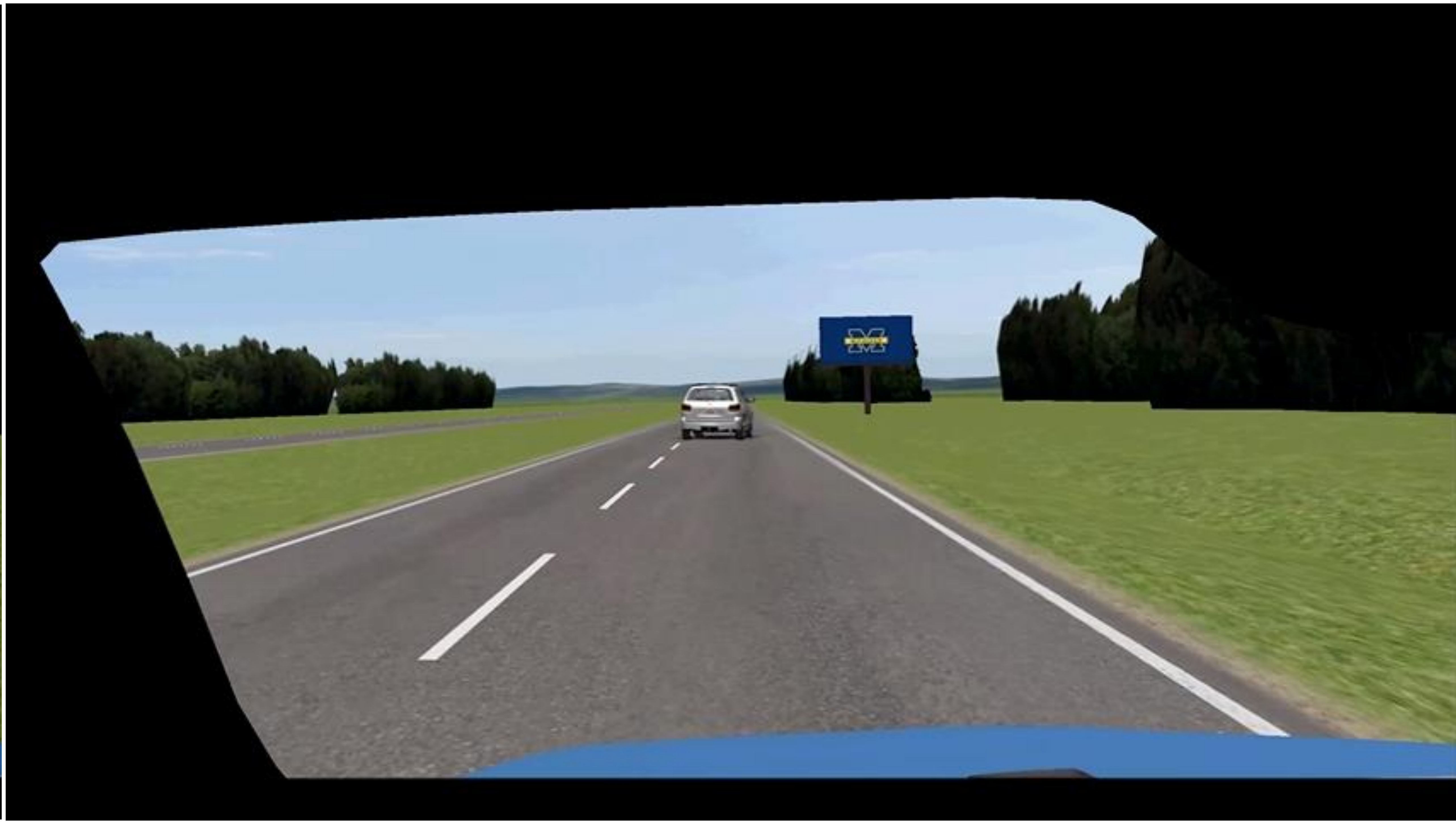


Naturalistic environment *vs* accelerated environment

Naturalistic Environment



Accelerated Environment



Zhao, "Accelerated Evaluation of Automated Vehicles Safety in Lane-Change Scenarios Based on Importance Sampling Techniques", IEEE ITS, 2017.

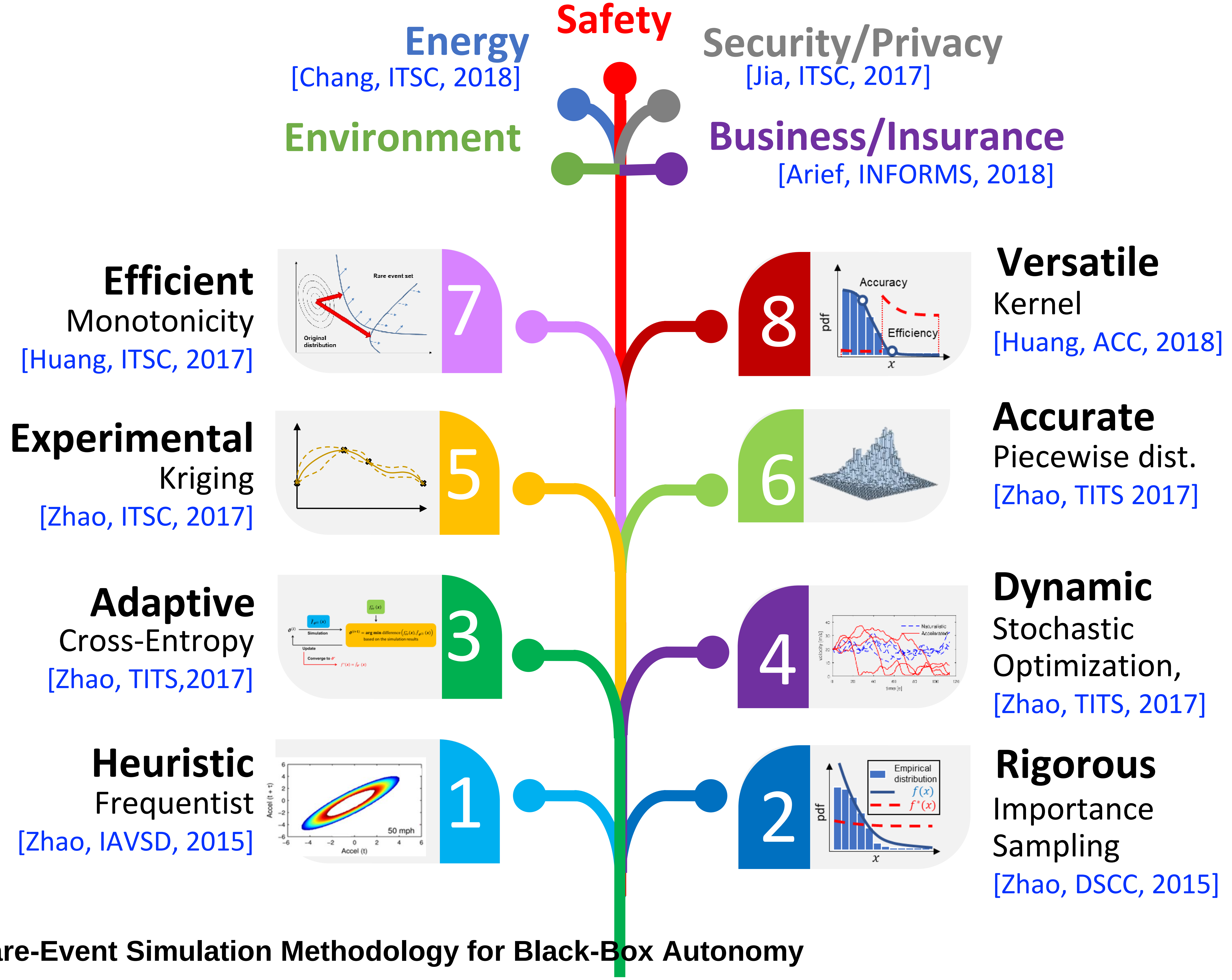
Accelerated Evaluation

Ongoing projects:

"Development of provable autonomous vehicle **evaluation approaches** with efficient data collection, unsupervised analysis, and high-dimensional stochastic models of on-road driving environment" (Uber, PI)

"Development of efficient multi-model **annotation and checking tools** based on synthesized learning methods" (Bosch, PI)

"Development of a "primary other **test vehicle**" for the testing and evaluation of high-level automated vehicles" (Toyota, Co-PI)

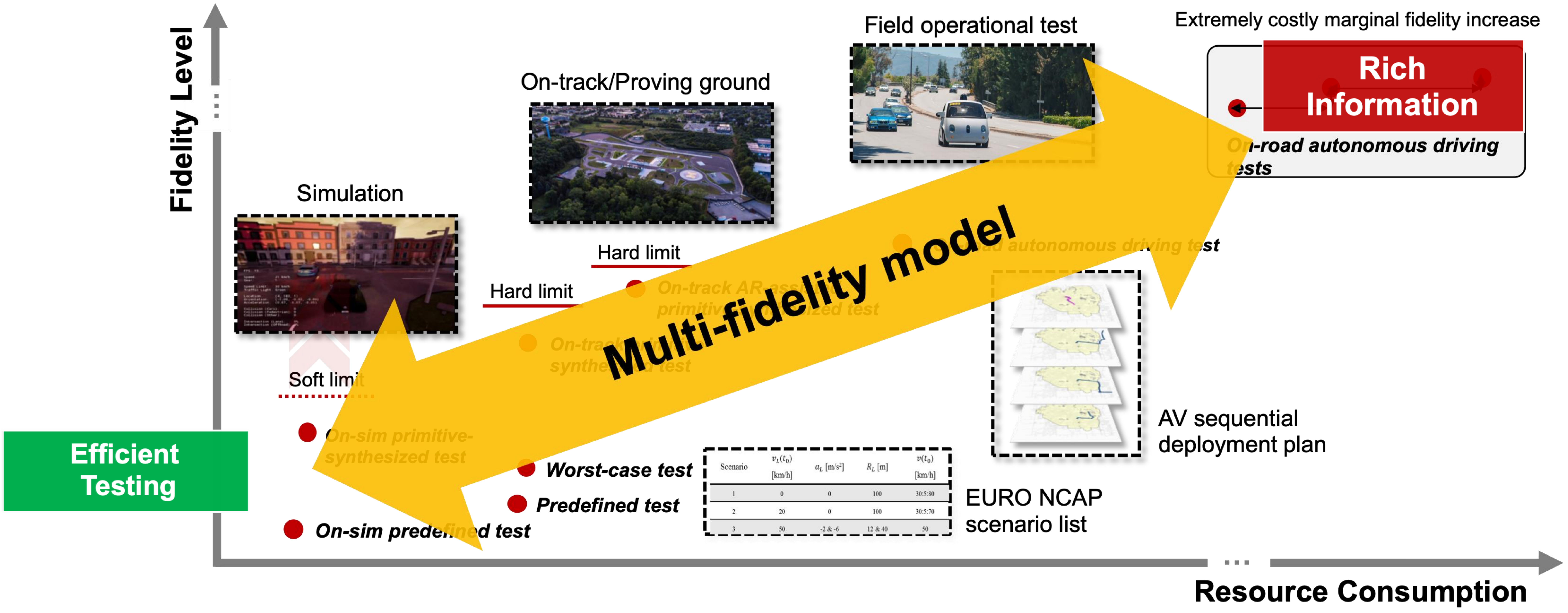


Arief, Deep Probabilistic Accelerated Evaluation (Deep-PrAE): A Certifiable Rare-Event Simulation Methodology for Black-Box Autonomy

Uber · TOYOTA RESEARCH INSTITUTE · BOSCH · Ford · SAIC 上汽集团 SAIC MOTOR · DENSO · NSF · Department of Transportation · Mobility21 A USDOT National University Transportation Center · traffic21 · M city University of Michigan

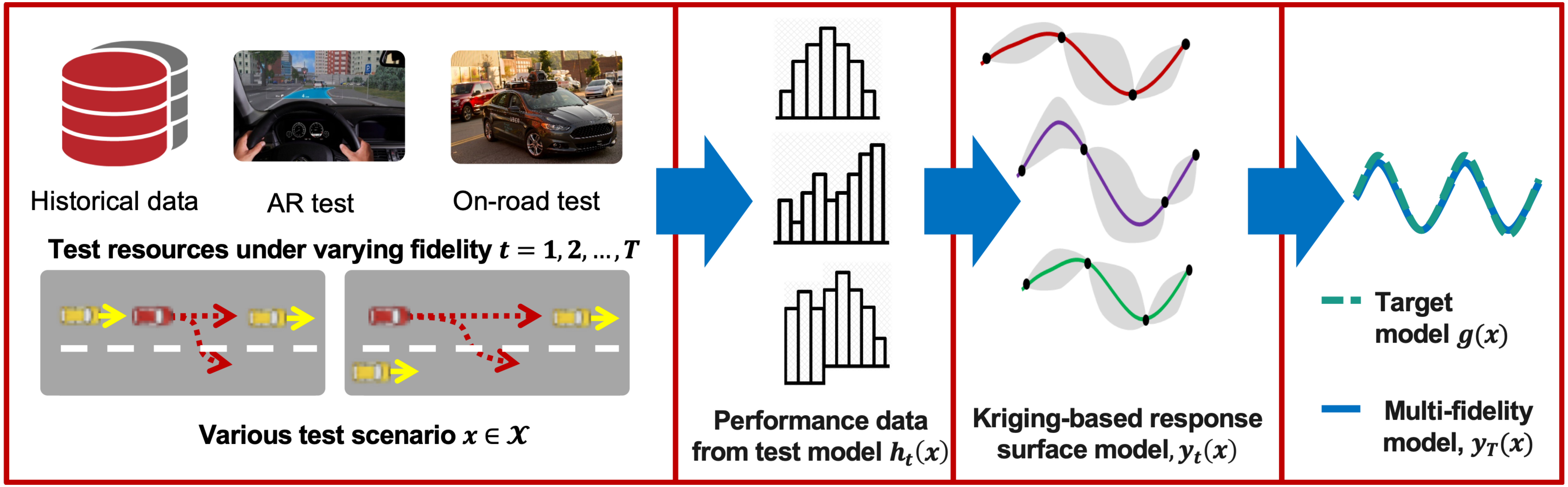
Accelerated data collection

- We propose the use of multi-fidelity models to synthesize high-efficiency and rich-information data collection



Huang "Synthesis of Different Autonomous Vehicles (AV) Test Approaches ", ITSC, 2018

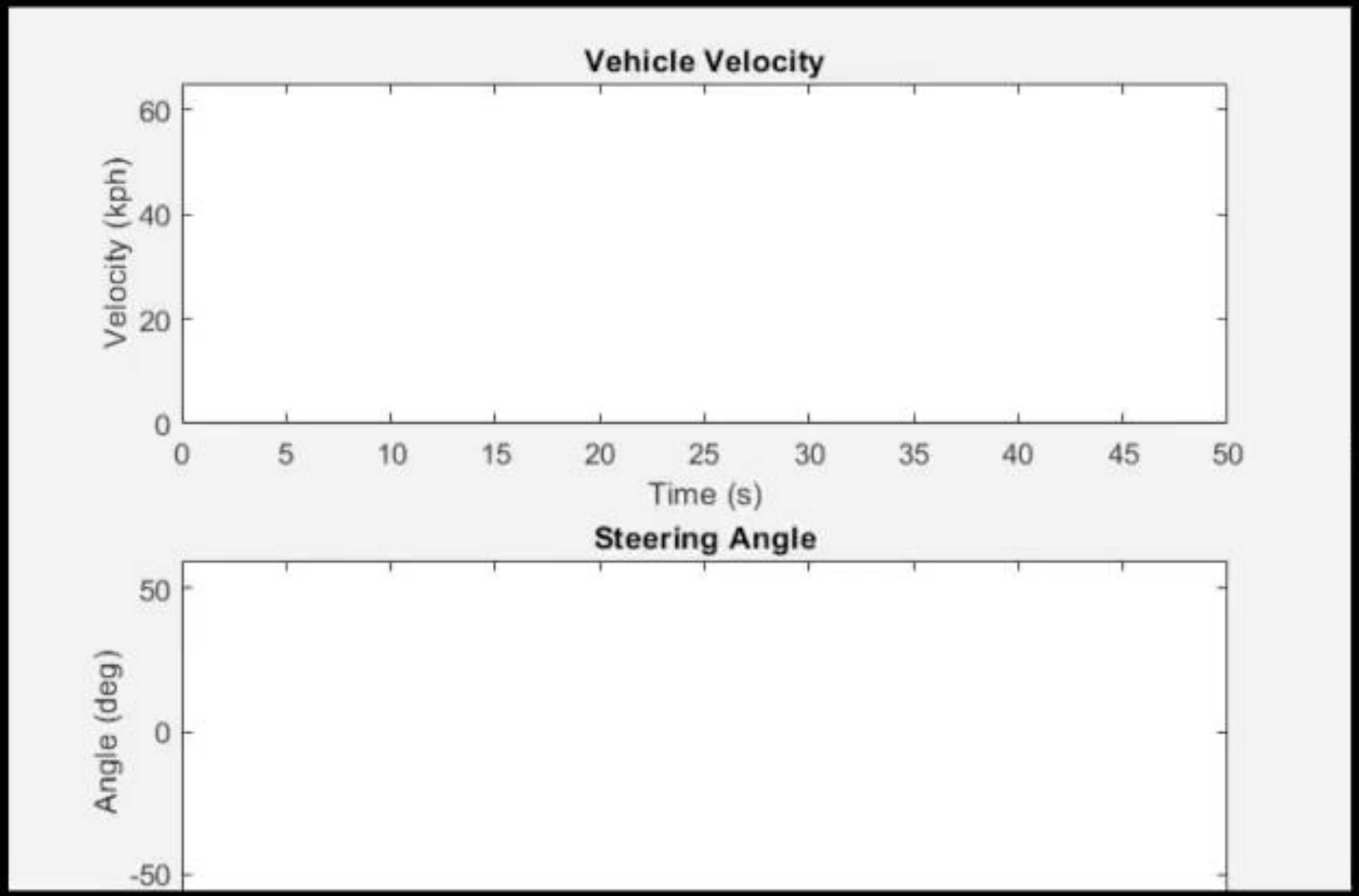
- Multi-fidelity models (e.g. Co-Kriging) are promising to synthesize information among various testing modes

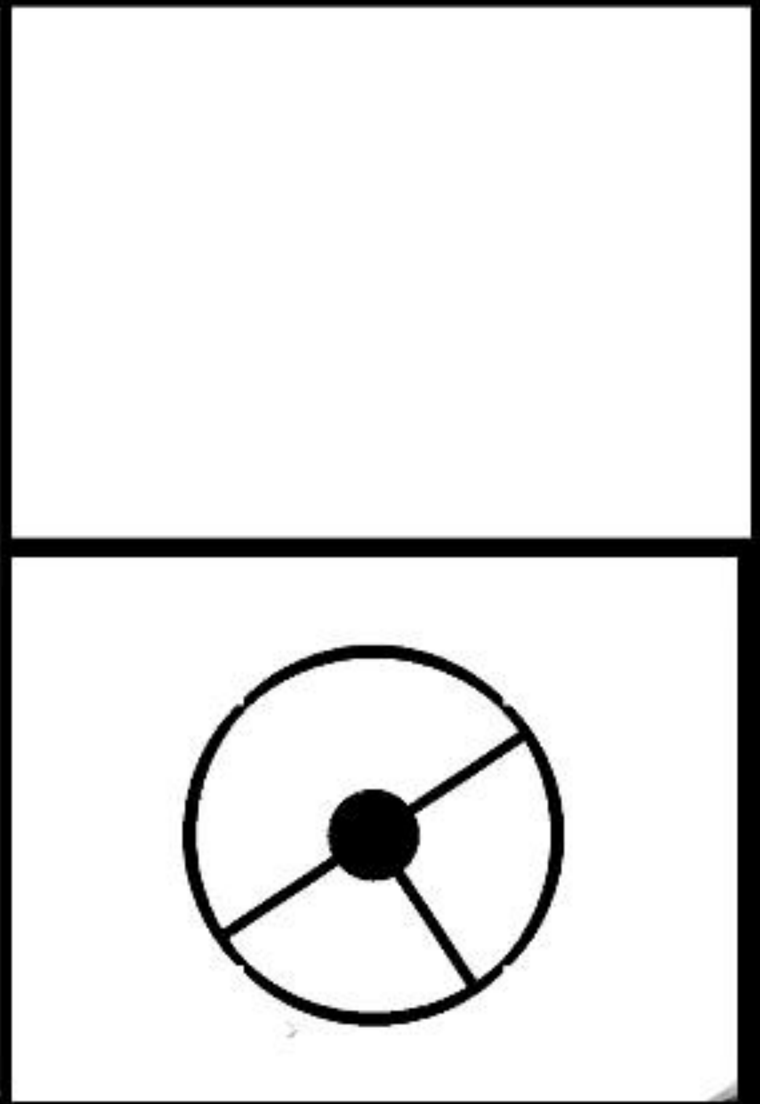


Huang "Synthesis of Different Autonomous Vehicles (AV) Test Approaches ", ITSC, 2018

Collect Data at Pittsburgh


Tepper School of Business
Heinz College of Information Systems
Gates Hillman Complex
Carnegie Mellon University Office of...
United States Postal Service
The Cut
The Mall
Wiegand Gymnasium
Gesling Stadium
Forbes Ave
Morewood Ave
Devon Rd
Penton Rd
Beeler


Vehicle Velocity
Velocity (kph)
60
40
20
0
0
5
10
15
20
25
30
35
40
45
50
Time (s)
Steering Angle
Angle (deg)
50
0
-50



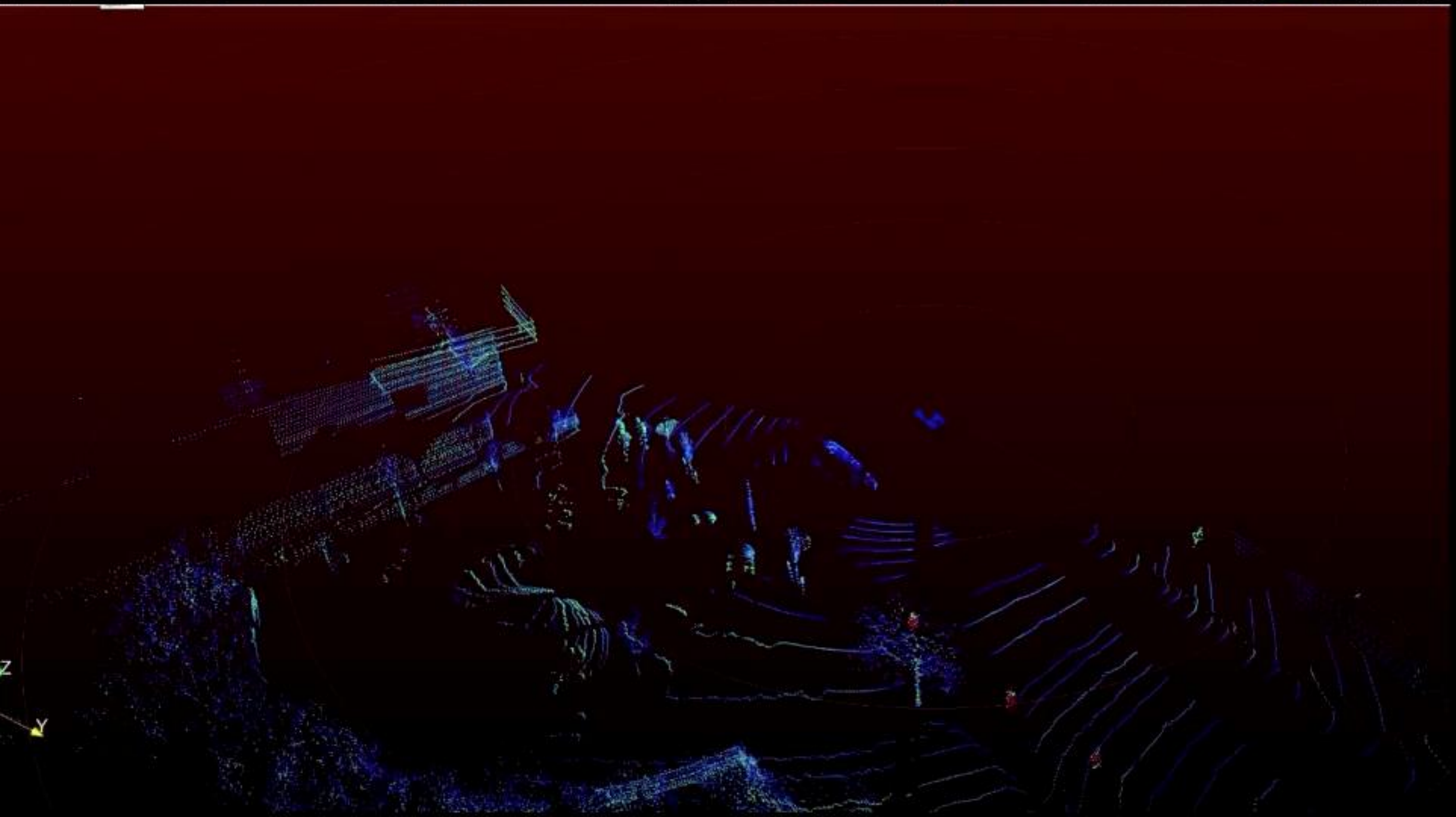

Z
Y


Image
Camera
Measure
2D Pose Estimate
2D Nav Goal
Publish Point
dynamic_reconfigure/Config
7 /camera/image_rect_color/compressedDepth/
dynamic_reconfigure/ConfigDescription
8 /camera/image_rect_color/compressedDepth/
dynamic_reconfigure/Config
9 /pandar_points
PointCloud2
10 /piksi/age_of_corrections
/AgeOfCorrections
11 /piksi/base_pos_ecef
/BasePosEcef
12 /piksi/baseline_ned
/BaselineNed
Add
Time

Not Secure — traffic-net.org

TRAFFICNET2.0 HOME DATASETS DOCUMENTATION TERMS CITATION CONTACT

Traffic Net 2.0

A one-stop platform for scenario-based self-driving database

Quick Access

Download

Sample Usage

Sensor Locations

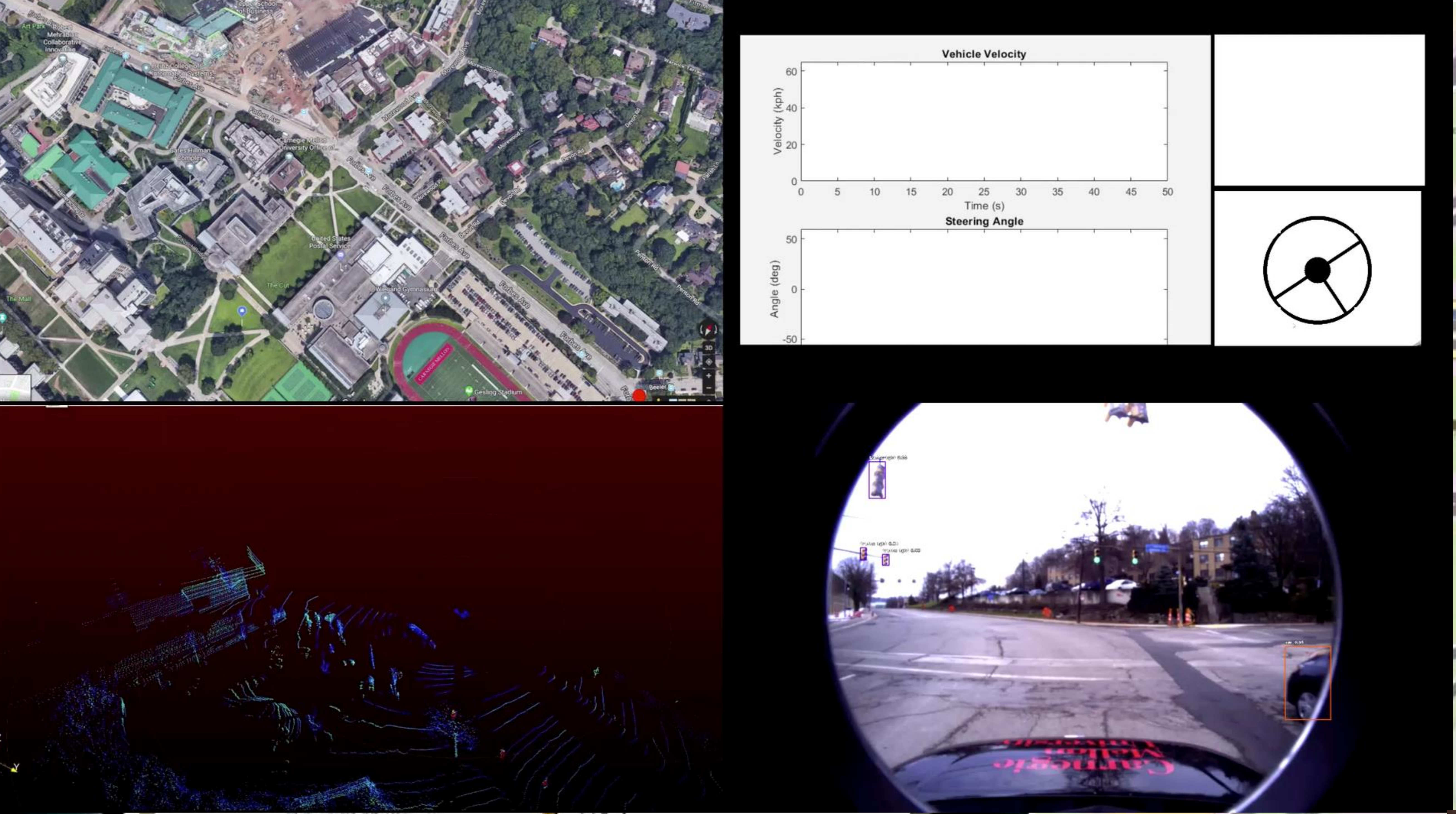
Dataset Discription

Data Format

Frequently Asked Questions

Datasets

[Zhu, Wang, Zhao, Integrating Heterogeneous Driving data For Autonomous Vehicles , ITSC, 2018]



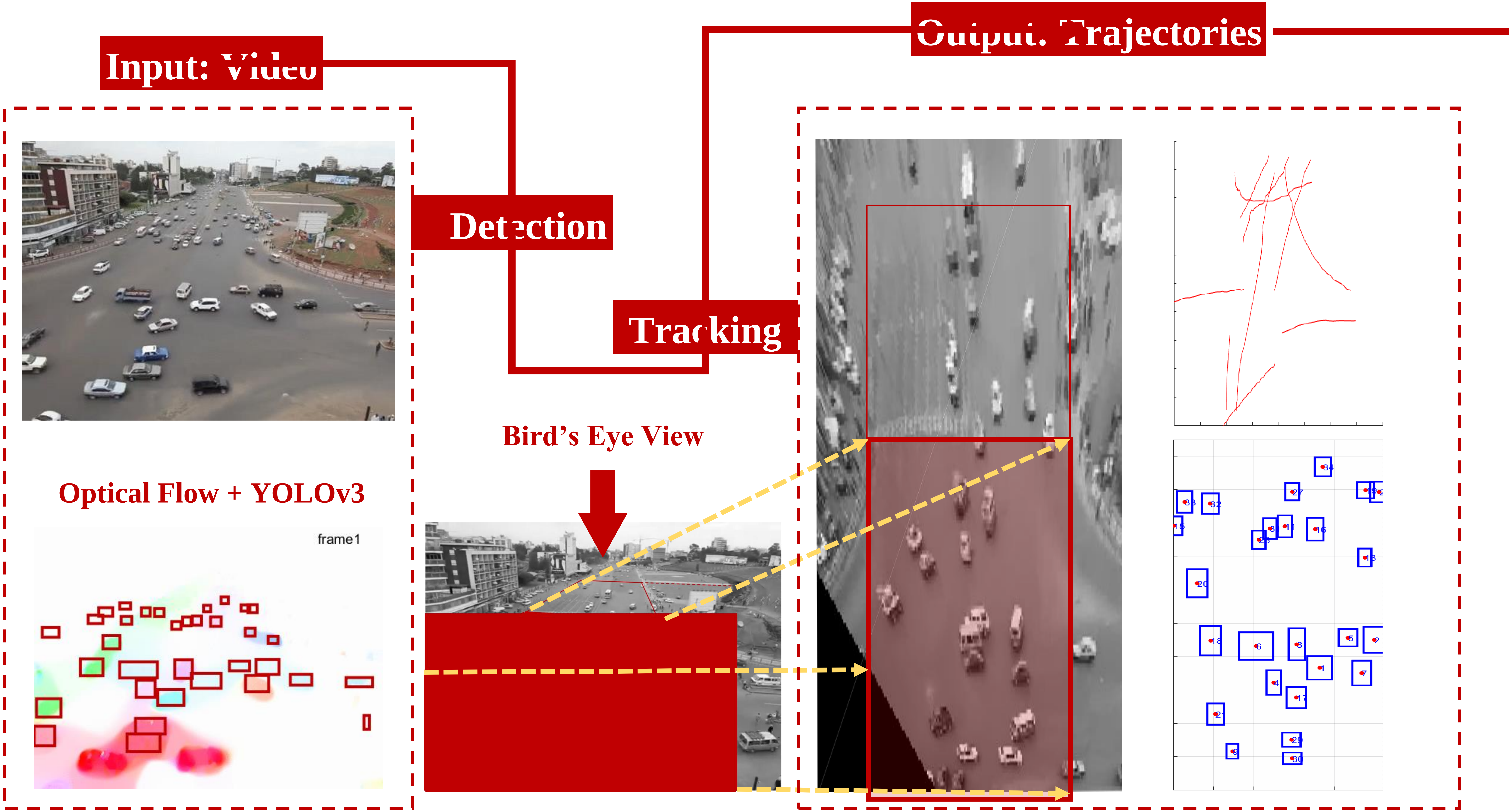
Liu, Feng, et al

Xu, et al

Guo, ITSC, 2019

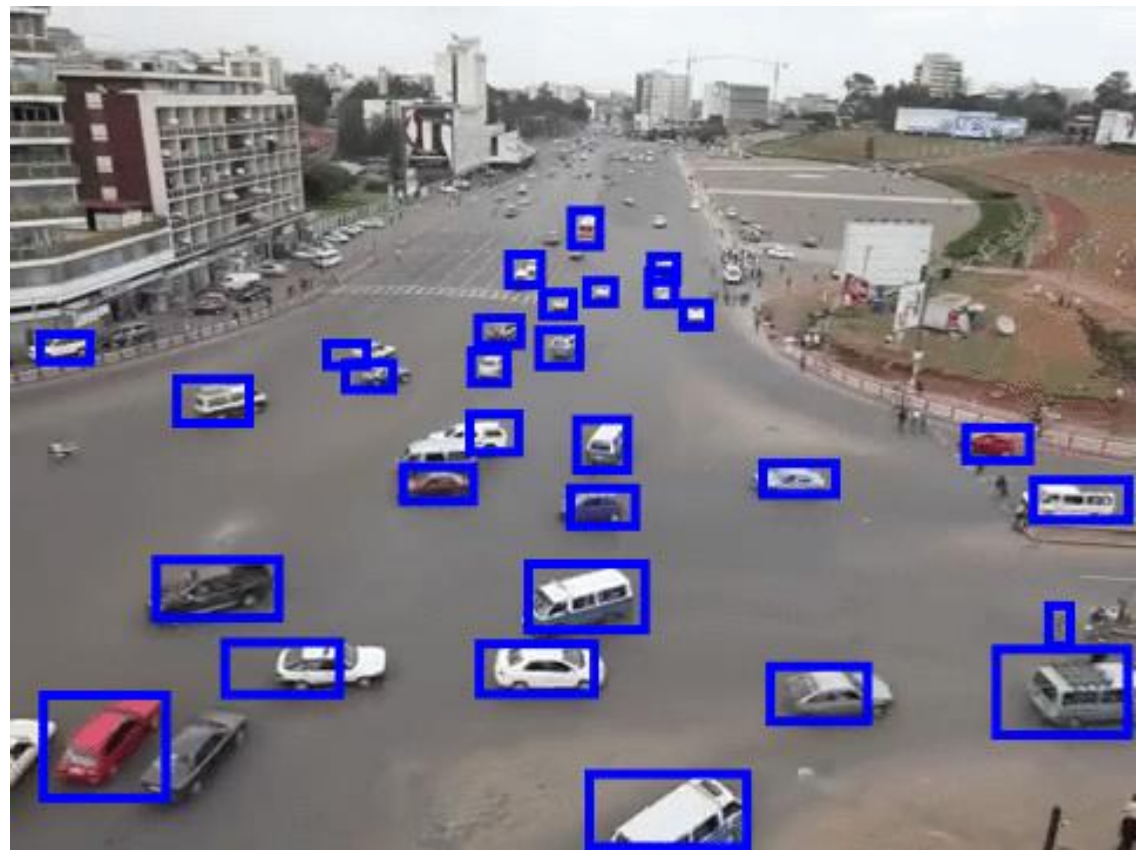
Endo Ethiopia

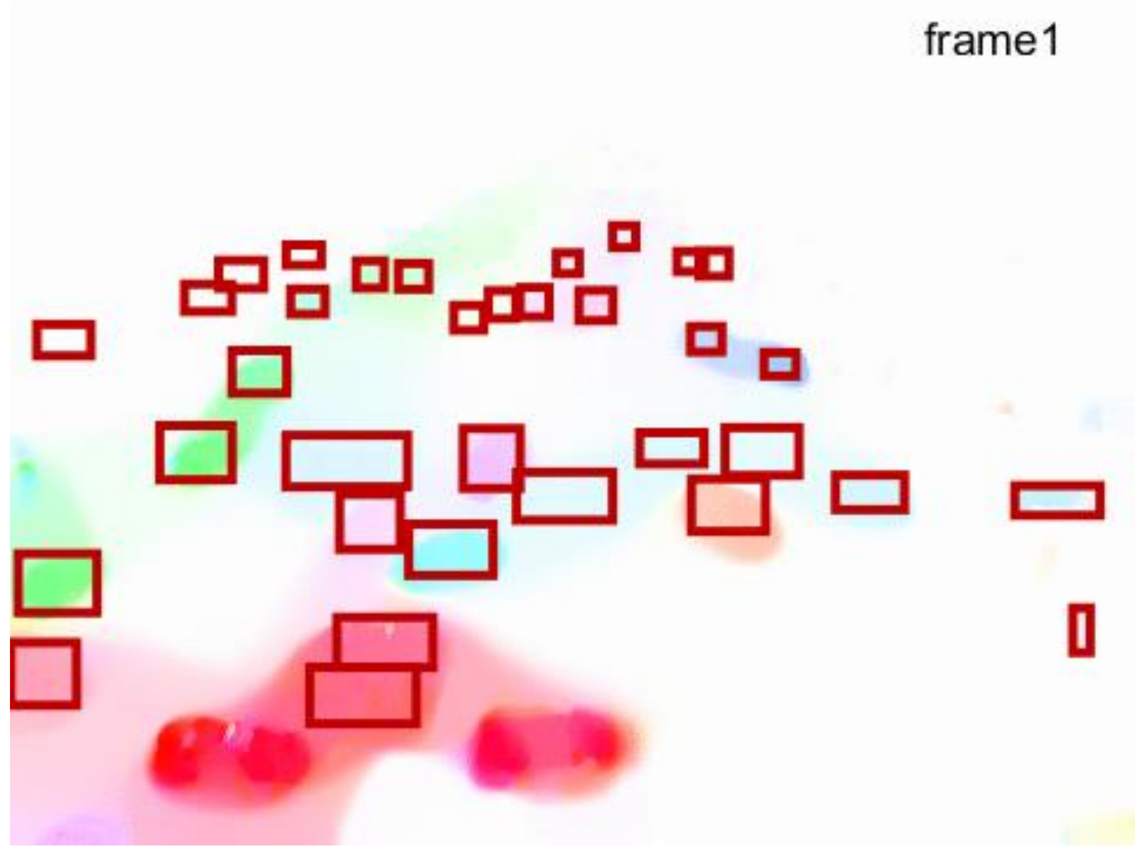
Viajes y Aventuras

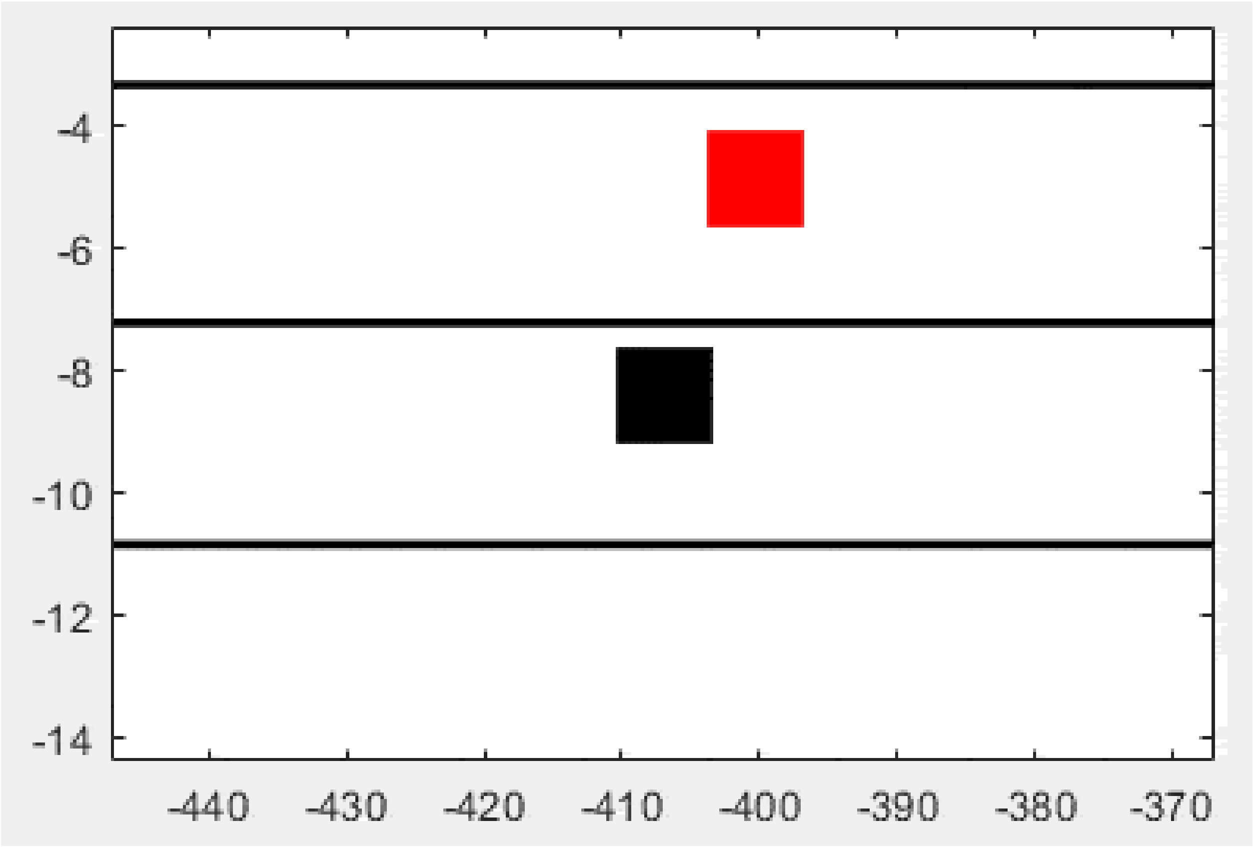


Input: Video
Detection
Tracking
Output: Trajectories
Optical Flow + YOLOv3
frame1
Bird's Eye View

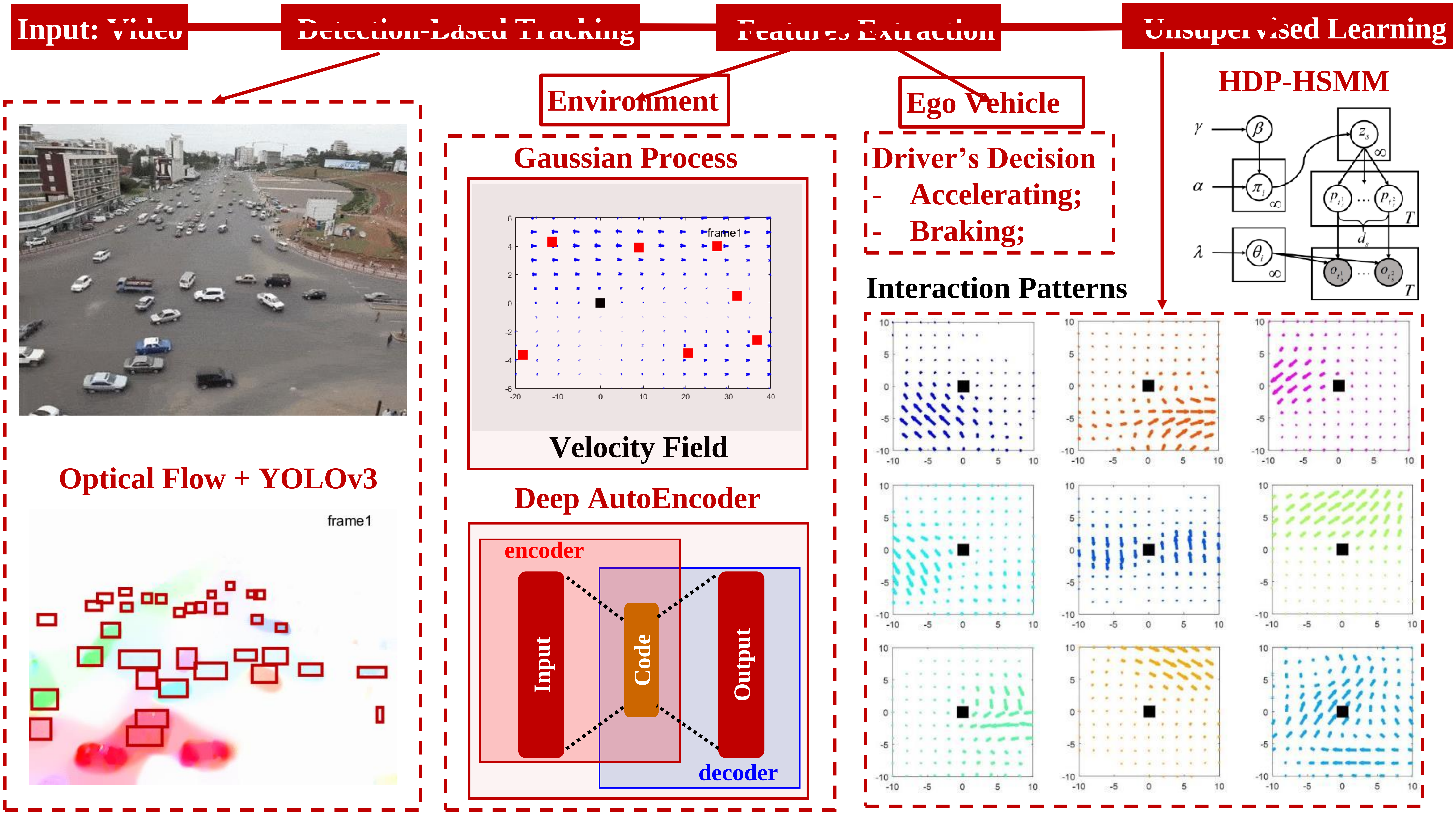
Trajectories
Features Extraction
Unsupervised Learning
Environment
Self State
Gaussian Process
frame1
Velocity Field
Convolutional Autoencoder
Low-Level Features
Encoder
Decoder
Driver's Decision
- Accelerating;
- Braking;
- Constant speed;
HDP-HSMM
C. Zhang, et al "A General Framework of Learning Multi-Vehicle Interaction Patterns from Video," ITSC, 2019
Multiple Vehicles Interaction Patterns
Clusters
1
(Accelerating)
2
(Accelerating)
3
(Braking)
4
(Constant speed)
5
(Braking)
6
(Accelerating)







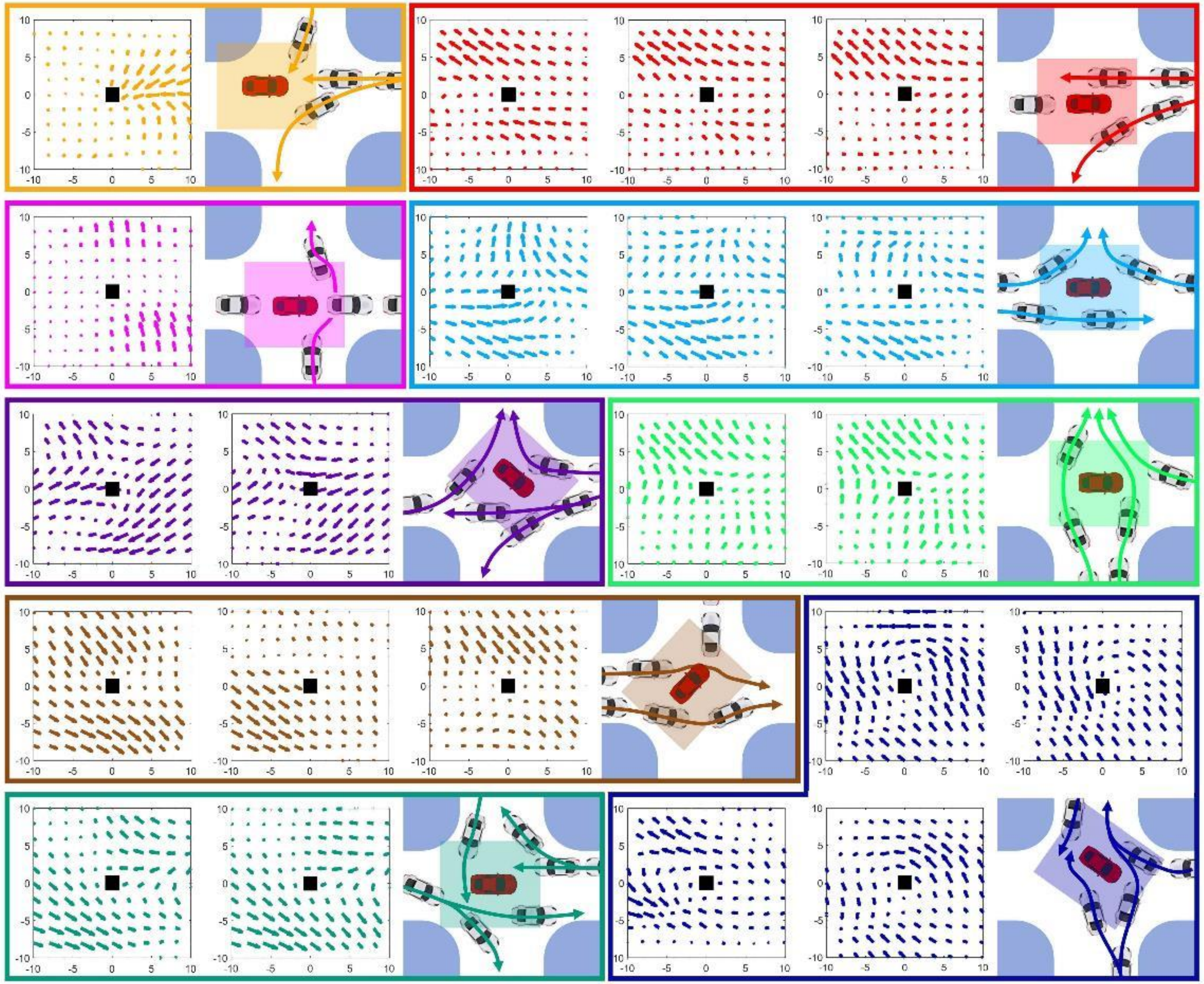
... the number of surrounding vehicles is time-varying …



Input: Video
Detection-Based Tracking
Features Extraction
Unsupervised Learning
Environment
Ego Vehicle
HDP-HSMM
Gaussian Process
frame1
Velocity Field
Driver's Decision
- Accelerating;
- Braking;
Interaction Patterns
Optical Flow + YOLOv3
frame1
Deep AutoEncoder
encoder
Input
Code
Output
decoder

Results - Traffic Primitives

Video results



Learning to Collide: An Adaptive Safety-critical Scenarios Generating Method

Wenhao Ding[1], Baiming Chen[2], Minjun Xu[1], Ding Zhao[1]

[1] Department of Mechanical Engineering, Carnegie Mellon University

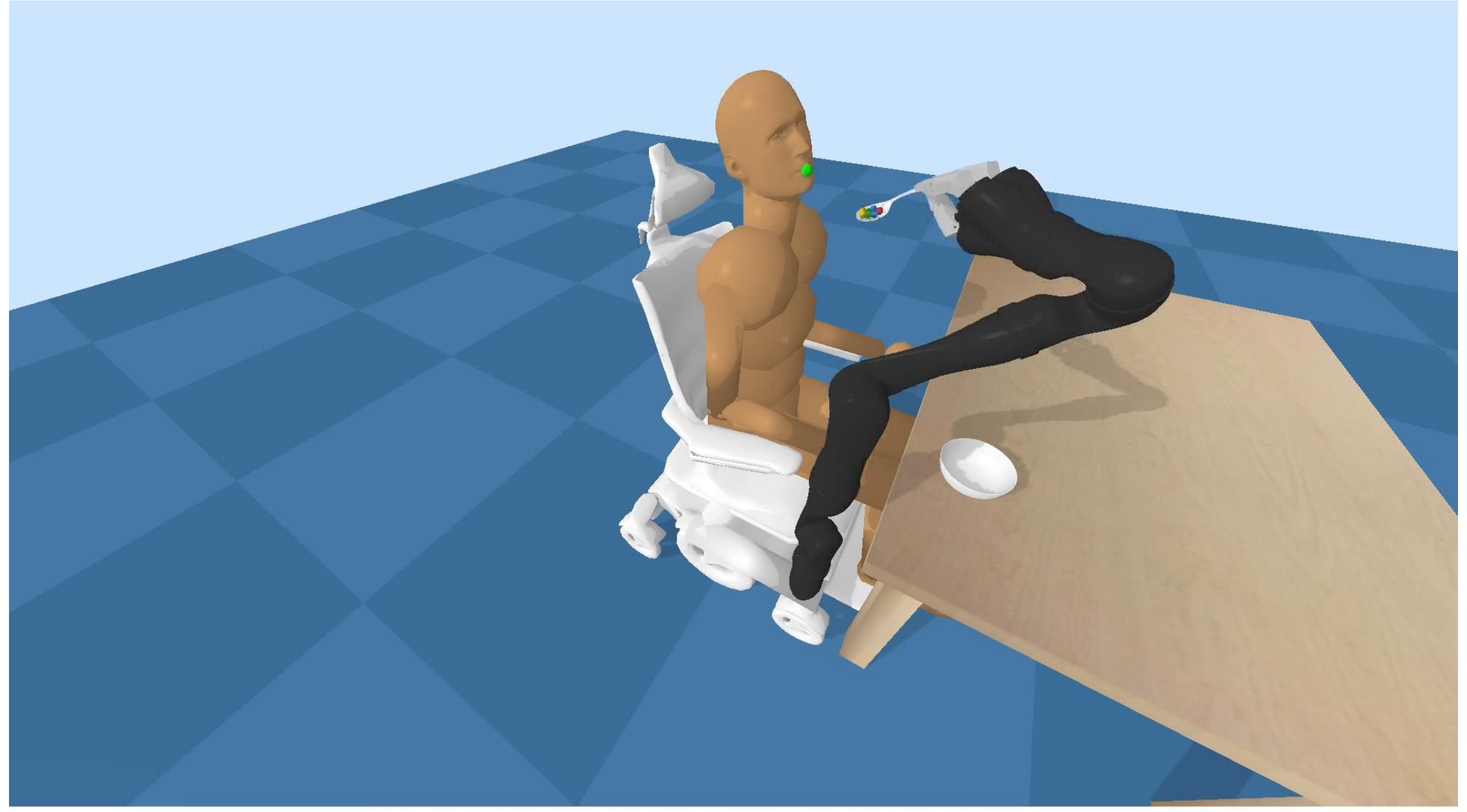
[2] Department of Automotive Engineering, Tsinghua University

Motivation

Big gap between RL and real-world applications:

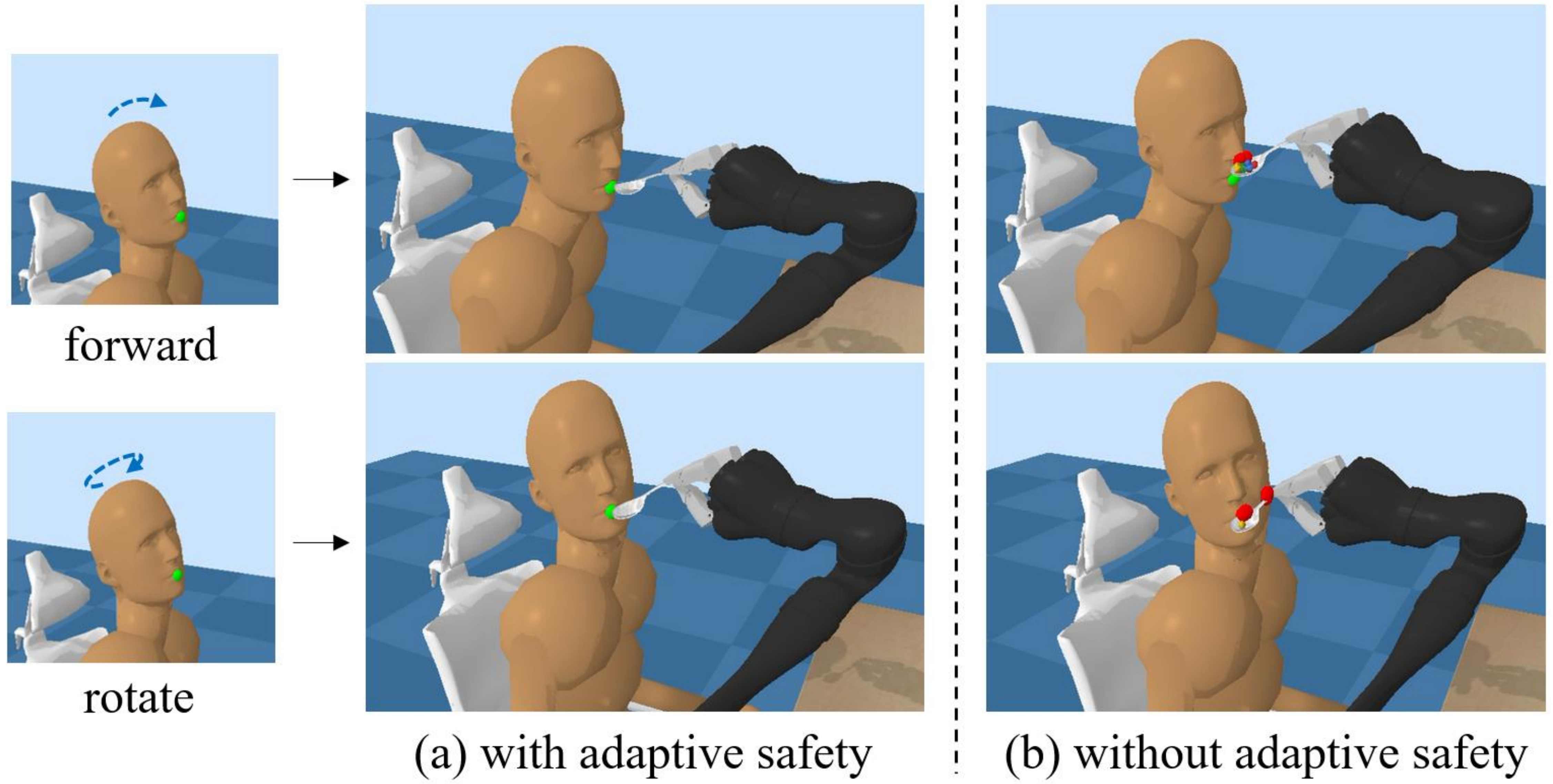
- Lack of safety guarantee
  - Most RL algorithms do not consider safety constraints during exploration.
- Lack of fast adaptability
  - Disturbances in the real-world: external forces, evolving opponents, action delays, etc.

without environment disturbance



[4] Erickson, Zackory, et al. "Assistive gym: A physics simulation framework for assistive robotics." arXiv preprint arXiv:1910.04700 (2019).

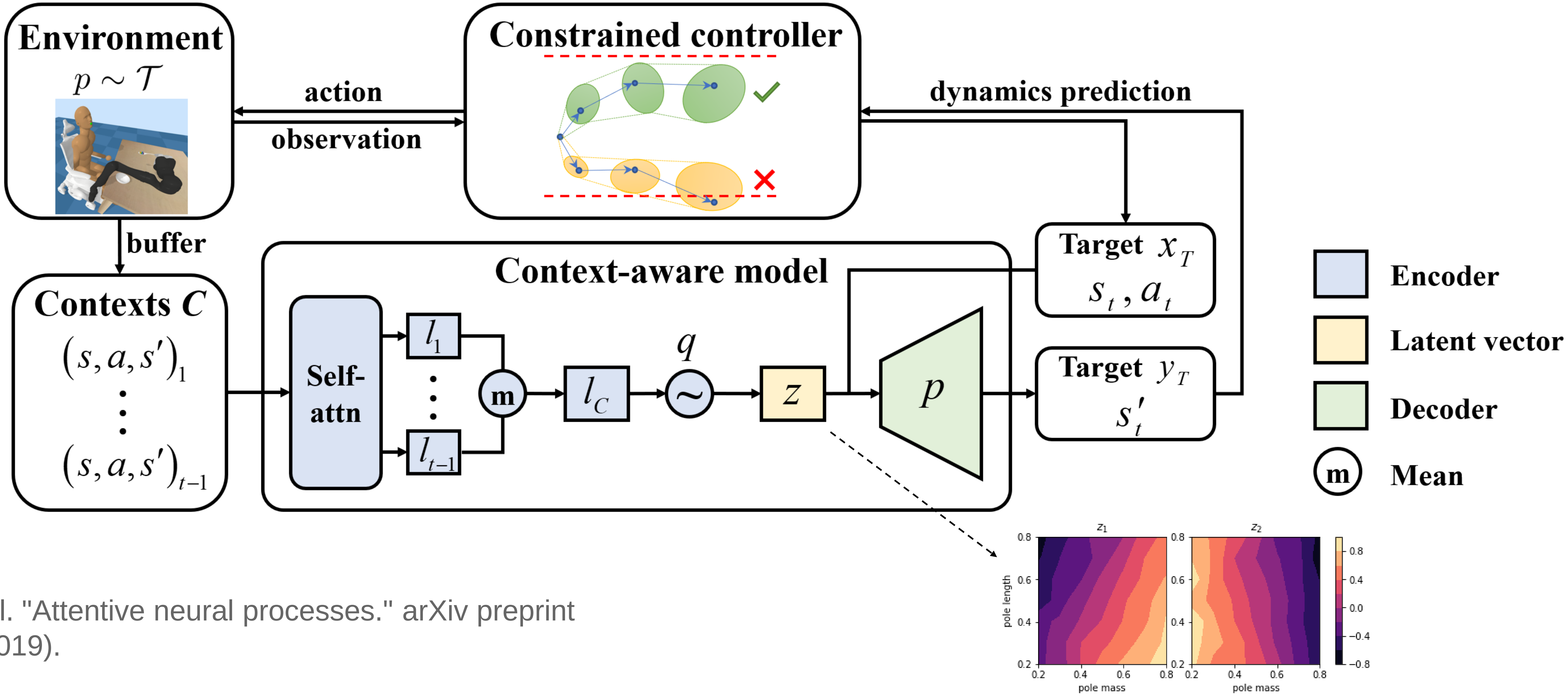
with environment disturbance



Modeling

Probabilistic context-aware model

- Encoder: disturbance identification based on the latest context data
- Decoder: dynamics prediction



[5] Kim, Hyunjik, et al. "Attentive neural processes." arXiv preprint arXiv:1901.05761 (2019).

Congratulations!

This is the last slide of Linear Control Systems

YOU MADE IT